NOISE REDUCTION IN 3D IMAGES USING MORPHOLOGICAL AMOEbas

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ABSTRACT
This article presents the use of morphological amoebas for the enhancement of 3D medical images. Morphological amoebas are kernels adapting their shape in such a way that they do not cross the contours of the image. They can be used in morphological operations in quite a similar way as classical kernel and are well-fitted for noise-reduction in 3D medical images.

Keywords: anisotropic filters, 3D image processing, morphological filters

1. INTRODUCTION
Anisotropic filtering such as pioneered by Perona and Malik ([1]) has emerged as a very effective tool for noise reduction as well as contour enhancement in images. Its effectiveness derives mostly from the fact that the filters take into account local information in order to adapt themselves to the content of the image: diffusion through a strong local gradient will be hampered so that firmly marked contours will neither be blurred nor displaced, which is a common problem in classical filtering.

Mathematical morphology has for some time offered an answer to the problem of contour displacement through the use of levelings ([8]) coupled to a noise reduction filter. A leveling has the interesting property of either preserving the position of a contour, or to erase it completely.

However, problems arise when strongly connected noise is connected to a contour: it is almost perfectly reconstructed, although it has been nearly wiped out by the first filtering (see figure 1).

This is often only a side issue, since for most 2D images it is the regions we are interested in. However, things are different with 3D medical images ([9]). The visualization algorithms often use local gradient information for the computation of the shading of the voxels and many applications, especially in the medical field, use mostly the contour information to visualize the various objects and as such are very vulnerable to noise on the contours (see figure 5).

Inspired by the anisotropic diffusion and the coupling of geometric and grayscale information ([5]) we have devised the morphological amoebas which are dynamic structuring elements (or rather structuring functions, [2]) that adapt their shapes to locally follow the contour lines and thus make it possible to use standard morphological operations with “smart” kernels that will not cross strong contour lines.

2. MORPHOLOGICAL AMOEbas
The construction of morphological amoebas and some of their properties are discussed in [7], so only a brief overview shall be provided here.

The classical square kernel of radius \( r \) is formally the ball of radius \( r \) associated with the \( L^\infty \) norm. Amoebas are defined in the same way, with a sort of geodesic distance instead of a purely geometric one.

Definition. Let \( d_{\text{pixel}} \) be a distance defined on the values of the image, for example a difference of gray-value, or a color distance.

Let \( \sigma = (x = x_0, x_1, \ldots, x_n = y) \) be a path between points \( x \) and \( y \). Let \( \lambda \) be a real positive number. The length of the path \( \sigma \) is defined as

\[
L(\sigma) = \sum_{i=0}^{n} 1 + \lambda d_{\text{pixel}}(x_i, x_{i+1})
\]
The “amoeba distance” with parameter \( \lambda \) is thus defined as:

\[
\begin{align*}
    d_\lambda(x, x) &= 0 \\
    d_\lambda(x, y) &= \min_{\sigma} \{ L(\sigma), x \neq y \}
\end{align*}
\]

This distance performs the required coupling between the geometric distance and the grayscale distance, with the possibility of adjusting the parameter \( \lambda \) to take more or less into account the gradient information.

The amoebas presented in this article are the balls defined by this distance, and thus have two parameters: the radius of the ball and the parameter \( \lambda \) which controls the strength of the gradient penalty. Note that for \( \lambda = 0 \) the amoeba is square-shaped (or hexagonal when using that type of connectivity).

Figure 2 presents the shape of an amoeba at various places over the an image, and figures 3 and 4 show the result of a morphological closing by a regular square kernel and an amoeba.

Fig. 2. Shape of an amoeba at various positions on a 2D image. Note that it reaches its maximal extension on flat zones, such as the background or the center of the disc, and it is reluctant to cross over high gradients at the edge of the disc.

Fig. 3. Closing of an image by a large structuring element. The structuring element does not adapt its shape and merges two distinct objects.

Fig. 4. Closing of an image by an amoeba. The amoeba does not cross the contour and as such preserves even the small canals.

3. APPLICATION TO 3D IMAGES

3.1. The transparency problem

When displaying directly 3D data (as opposed to studying a stack of 2D images), it is essential that the user be able to see the objects they are interested in. This is why most modern renderers include a complex transparency and shading model that makes it possible to peek far inside the image to display the interesting objects. To make a quantitative analysis easier, a synthetic image was created that presents many similarities with 3D scanner images, especially images of the cardiac region: strong textures and thin vessels to preserve.

Figure 5 illustrates the problem due to strong noise in a 3D image and figure 6 shows that levelings are ineffective as even though most noise is removed inside the objects, the noise on the contours is well reconstructed. However, as with most morphological tools, amoebas are adimensional and can be used without modification on 3D images.

Fig. 5. A synthetic 3D volume presenting many similarities with medical images of the cardiac regions. Left: original image. Right: image with addition of noise.
Although the median filters much noise away, the reconstruction (needed to recover the position of the contours) reconstructs most of the noise on the borders of the object, removing most of the transparency.

3.2. Median filtering with amoebas

One important aspect of the filtering of such medical images is that those images are monospectral: coloring and, ultimately, tissue identification is done using a look-up table. This means that a shifting of the values may have a dramatic effect on the visualization and consequently on the interpretation of the images. This is why we have chosen to first test the median: with its property of returning only values existing in the image, the median lessens the risk of misinterpretation. However, traditional median filtering does not preserve well the contours, and may remove small details which may be crucial to a physician’s analysis, hence the use of amoebas.

For each pixel the processing is done in two steps: first compute the shape of the amoeba centered on the pixel and then sample the values of the pixel inside the amoeba, feed them to the median operator and write the result at the center of the amoeba in the output image.

4. COMPLEXITY AND OPTIMIZATION

4.1. Theoretical complexity

The theoretical complexity of a simple amoeba-based filter (erosion, dilation, mean, median) can be asymptotically approximated by:

\[ T(n, k, op) = O \left( n \ast \left( op(k^d) + amoeba(k, d) \right) \right) \]

Where \( n \) is the number of pixels in the image, \( d \) is the dimensionality of the image (usually 2 or 3), \( k \) is the maximum radius of the amoeba, \( op(k^d) \) is the cost of the operation and \( amoeba(k, d) \) is the cost of computing the shape of the amoeba for a given pixel.

The shape of the amoebas is computed by a common region-growing implementation using a priority queue. Depending on the priority queue used, the complexity of this operation is in slightly more than \( O(k^d) \) (see [3] and [4] for advanced queueing data structures).

Therefore, for erosion, dilation or mean as operators, we have a complexity of a little more than \( O(n \ast k^d) \) which is the complexity of a filter on a fixed-shape kernel. It has indeed been verified in practice that, while being quite slower than with fixed-shape kernels (especially optimized ones), filters using amoebas tend to follow rather well the predicted complexity, and do not explode. Computation times on a modern machine (Pentium 4 2GHz) without heavy optimizations may take up to an hour for images of size 256 × 256 × 256 and amoebas with a maximum radius of up to 5
4.2. Simple optimizations

An important fact to take into account is that the radius parameter is like an amount of energy given to the amoeba. It can be used either to climb slopes (with a penalty given by the $\lambda$ parameter) or it can be used to expand in flat areas. This amount of energy needs to be quite high so that the amoeba can jump over noisy pixels (though not too high so that it does not cross too much over strong gradient lines). However such a high energy means that in flat areas the amoeba will grow to a very large size, which means that not only will the shape be costly to compute but the resulting sample of pixel values will be quite large and so the filter operator will be accordingly long. A very simple yet dramatically effective optimization is thus to impose an upper bound on the size of the amoeba, the value of which depends on the type of noise and the characteristic size of the image elements. This can reduce the cost of computation by an order of magnitude without any detectable loss of effectiveness.

Another form of optimization is to compute the shape of the amoeba on a slightly filtered version of the original image, such as a Gaussian filtering. This will reduce small noise without moving too much the contours and enable the use of smaller, less energetic amoebas. See [7] for a discussion on prefiltering and iteration of this process.

5. CONCLUSION AND FUTURE WORK

The filtering of 3D images by morphological amoebas, though still in its infancy, seems very promising. Expressing the coupling between image data and geometry through a kernel makes it possible to implement a much larger range of filters to an image than was possible before. Common filters such as median and mean show excellent results, as well as rank filters. Alternate sequential filters also show promising results, especially on very contrasted images. Furthermore, using the amoeba to define the neighborhood of each pixel makes it possible to use it in other morphological algorithms such as the watershed, with a greater degree of control of the flooding process.

Another area where improvement is to be expected is the computation of the shape of the amoeba. Not only should it be possible to use elaborate gradient estimation such as proposed in [6], but also providing more complex behaviors for the amoebas, such as an incompressible minimum element, to guarantee at least some diffusion, or on the contrary a minimum size requirement to prevent diffusion through small holes.

All these possible improvement should be quite easy to incorporate to the amoeba framework and can readily be made available to many types of filters.

6. ACKNOWLEDGMENTS

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7. REFERENCES


