Symmetric and Semisymmetric Graphs Construction Using G-graphs

Alain Bretto *
Université de Caen, GREYC
CNRS UMR-6072, Campus II
Bd Marechal Juin BP 5186,
14032 Caen cedex
Caen, France
alain.bretto@info.unicaen.fr

Luc Gillibert
Université de Caen, GREYC
CNRS UMR-6072, Campus II
Bd Marechal Juin BP 5186,
14032 Caen cedex
Caen, France
lgillibe@info.unicaen.fr

Bernard Laget
Ecole Nationale d’Ingénieurs
de Saint-Etienne, DIPI
58, rue Jean Parot, 42023
Saint-Etienne Cedex 02.
France.
laget@enise.fr

ABSTRACT
Symmetric and semisymmetric graphs are used in many scientific domains, especially parallel computation and interconnection networks. The industry and the research world make a huge usage of such graphs. Constructing symmetric and semisymmetric graphs is a large and hard problem. In this paper a tool called G-graphs and based on group theory is used. We show the efficiency of this tool for constructing symmetric and semisymmetric graphs and we exhibit experimental results.

Categories and Subject Descriptors
G.2.2 [Discrete Mathematics]: Graph Theory—Graph algorithms

General Terms
Algorithms, Theory

Keywords
Symmetric graphs, semisymmetric graph, graphs from group, G-graphs.

1. INTRODUCTION
A graph that is both edge-transitive and vertex-transitive is called a symmetric graph. Such graphs are used in many domains, for example, the interconnection network of SIMD computers. But to construct them is a very hard problem. Usually, the graphical representation of groups is used for the construction of those graphs. A popular representation of a group by a graph is the CAYLEY representation. A lot of work has been done about these graphs [3, 10]. CAYLEY graphs have very nice highly-regular properties. But CAYLEY graphs are always vertex-transitive, and that can be a limitation. In this article we present and use G-graphs introduced in [4, 5]. G-graphs, like CAYLEY graphs, have highly-regular properties, consequently G-graphs are a good alternative tool for constructing some symmetric graphs. After the definition of these graphs we give a characterization of bipartite G-graphs. Then, using this characterization, we build a powerful algorithm based on G-graphs for computing symmetric graphs.

The classification of symmetric graphs is a very interesting problem. Ronald M. Foster started collecting specimens of small cubic symmetric graphs prior to 1934, maintaining a census of all such graphs. In 1988 the current version of the census was published in a book containing some graphs up to the order 512 [7]. But symmetric graphs are not the only interesting graphs. There exist regular graphs which are edge-transitive but not vertex-transitive [6], they are called semisymmetric graphs, and it is quite difficult to construct them [14, 11]. Indeed, CAYLEY graphs are always regular and vertex-transitive, so they cannot be semisymmetric, but G-graphs can be either regular or non-regular, vertex-transitive or not vertex-transitive. In this paper we exhibit an efficient algorithm, based on G-graphs, constructing cubic semisymmetric graphs. So, with G-graphs, it becomes easy not only to extend the Foster Census up to order 800, but also to construct cubic semisymmetric graphs, quartic symmetric and semisymmetric graphs, quintic symmetric and semisymmetric graphs and so on.

2. BASIC DEFINITIONS
We define a graph \( \Gamma = (V; E; \epsilon) \) as follows:

- \( V \) is the set of vertices and \( E \) is the set of edges.
- \( \epsilon \) is a map from \( E \) to \( P_2(V) \), where \( P_2(V) \) is the set of subsets of \( V \) having 1 or 2 elements.

In this paper graphs are finite, i.e., sets \( V \) and \( E \) have finite cardinalities. For each edge \( e \), we denote \( \epsilon(a) = [x; y] \) if \( \epsilon(a) = \{x, y\} \) with \( x \neq y \) or \( \epsilon(a) = \{x\} \) if \( x = y \). If \( x = y \), \( e \) is called loop. The set \( \{a \in E, \epsilon(a) = [x; y]\} \) is called multiedge or \( p \)-edge, where \( p \) is the cardinality of the set. We define the degree of \( x \) by \( \deg(x) = \text{card}\{a \in E, x \in a\} \).
3. GROUP TO GRAPH PROCESS

Let \((G, S)\) be a group with a set of generators \(S\). For any \(s \in S\), we consider the left action of the subgroup \(H = \langle s \rangle\) on \(G\). Thus, we have a partition \(G = \bigsqcup_{e \in T_s} (s)x\), where \(T_s\) is a right transversal of \(s\). The cardinality of \(s\) is \(|s|\) where \(O(s)\) is the order of the element \(s\). Let us consider the cycles

\[
(s)x = (x, sx, s^2x, \ldots, s^{o(s)-1}x)
\]

of the permutation \(g_s : x \mapsto sx\). Notice that \((s)x\) is the support of the cycle \((s)\). One cycle of \(g_s\) contains the unit element \(e\), namely \(s e = (e, s, s^2, \ldots, s^{o(s)-1})\). We now define a new graph denoted by \(\Phi(G; S) = (V; E; \epsilon)\) as follows:

- The vertices of \(\Phi(G; S)\) are the cycles of \(g_s, s \in S\), i.e., \(V = \bigsqcup_{e \in S} V_e\) with \(V_e = \{(s)x | x \in T_s\}\).
- For all \((s)x, (t)y \in V\), \(\{s\}x, \{t\}y\) is a \(p\)-edge if:
  \[
  \text{card}(\langle s \rangle x \cap (t)y) = p, \quad p \geq 1
  \]

Thus, \(\Phi(G; S)\) is a \(k\)-partite graph and any vertex has a \(O(s)\)-loop. We denote \(\Phi(G; S)\) the graph \(\Phi(G; S)\) without loops. By construction, one edge stands for one element of \(G\). We can remark that one element of \(G\) labels several edges. Both graphs \(\Phi(G; S)\) and \(\Phi(G; S)\) are called graph from a group or \(G\)-graph and we say that the graph is generated by the group \((G; S)\). Finally, if \(S = G\), the \(G\)-graph is called a

A graph \(G\) is called a

Theorem 3.1 (Algorithmic procedure)

The following algorithm constructs a \(G\)-graph from the list \(L\) of the cycles of a group:

\[
\text{Group_to_graph}_G(L)
\]

1. Add \(s\) to \(V\) for all \(s\) in \(L\)
2. For all \(s'\) in \(L\) for all \(x\) in \(s\) for all \(y\) in \(s'\)
   - if \(x = y\) then Add \((s, s')\) to \(E\)

For the construction of the cycles we use the following algorithm, written in the GAP programming language [12]:

\[
\text{InstallGlobalFunction (}
\]

\[
\text{c_cycles, function (G, ga)}
\]

\[
\text{local ls1, ls2, ga, k, x, o, a, res, G2;}
\]

\[
\text{res:=[];}
\]

\[
\text{G2:=List(G);}
\]

\[
\text{for a in ga do}
\]

\[
\text{gs:=[];}
\]

\[
\text{oa:=Order(a)-1;}
\]

\[
\text{ls2:=Set([]);}
\]

\[
\text{for x in G do}
\]

\[
\text{if not(x in ls2) then}
\]

\[
\text{ls1:=[];}
\]

\[
\text{for k in [0..oa] do;}
\]

\[
\text{Add(ls1, Position(G2, (a^k)*x));}
\]

\[
\text{AddSet(ls2, (a^k)*x);}
\]

\[
\text{od;}
\]

\[
\text{Add(gs, ls1);}
\]

\[
\text{fi;}
\]

\[
\text{od;}
\]

\[
\text{Add(res, ga);}
\]

\[
\text{od;}
\]

\[
\text{return res;}
\]

4. PROPERTIES OF THE G-GRAPHS

We now introduce some useful results:

**Proposition 1.** Let \(\Phi(G; S) = (V; E; \epsilon)\) be a \(G\)-graph. This graph is connected if and only if \(S\) is a generator set of \(G\).

**Proof.** If \(\text{card}(S) = 1, G = \langle s \rangle\). The graph has only one vertex. It is connected. Assume that \(\text{card}(S) \geq 2\). Let \((s)x\) \(\in V_s\) and \((s')y\) \(\in V_{s'}\), because \(G = \langle S \rangle\), there exists \(s_1s_2s_3 \ldots s_n \in S\) such that \(y = s_1s_2s_3 \ldots s_n x\).

\[
\begin{align*}
  x & \in \langle s \rangle x \cap \langle s_n \rangle x \\
  s_n x & \in \langle s_n \rangle x \cap \langle s_{n-1} \rangle s_n x \\
  s_{n-1}s_n x & \in \langle s_{n-1} \rangle s_n x \cap \langle s_{n-2} \rangle s_{n-1}s_n x \\
  \cdots
\end{align*}
\]
\[ s_2 \ldots s_n x \in \langle s_2 \rangle s_3 \ldots s_n x \cap \langle s_1 \rangle s_2 \ldots s_n x \]
\[ y \in \langle s_1 \rangle s_2 \ldots s_n x \cap \langle s' \rangle y \]

Consequently there exists a chain from \((s)x \in V_x \) to \((s')y \in V_y\). So \(\Phi(G; S)\) is a connected graph.

Conversely, let \(x \in G\). There exists \(s_1 \in S\) and \(x_1 \in T_i\) such that \(x \in (s_1) x_1\), with \(x = s_1 x_1\). The graph is connected so there exists a chain from \((s_1)x_1\) to \((s_1)\epsilon\):
\[ x = s_1^{k_1} x_1, \quad x_1 = s_2^{k_2} x_2, \ldots, \quad x_{k-1} = s_k^{k_k} x_k \]

With \(x_k = \epsilon\), so \(x = s_1^{k_1} s_2^{k_2} \ldots s_k^{k_k}\). □

**Proposition 2.** Let \(h\) be a morphism between \((G_1, S_1)\) and \((G_2, S_2)\), then there exists a morphism, \(\Phi(h)\), between \(\Phi(G_1; S_1)\) and \(\Phi(G_2; S_2)\).

**Proof.** We define \(\Phi(h) = \phi = (f, f^\#)\) in the following way:
- \(f: \cup_{s \in S_1} V_1, s \longrightarrow \cup_{s \in S_2} V_2, s\)
  \((s)x \longrightarrow (h(s))h(x)\)
- \(f^\#: E_1 \longrightarrow E_2\)
  \(([(s)x, (t)y); u] \longrightarrow \{[(h(s))h(x), (h(t))h(y)]; h(u)\}\)

It is easy to verify that \(\phi\) is a morphism from \(\Phi(G_1; S_1)\) to \(\Phi(G_2; S_2)\). So, any group morphism gives rise to a graph morphism. □

For abelian groups we have the following:

**Theorem 1.** Let \(G_1\) and \(G_2\) be two abelian groups. These two groups are isomorphic if and only if \(\Phi(G_1; G_1)\) and \(\Phi(G_2; G_2)\) are isomorphic.

**Proof.** It has been proved that group isomorphism leads to graph isomorphism. Now, suppose that \(\Phi(G_1; G_1)\) is isomorphic to \(\Phi(G_2; G_2)\). These two graphs have the same degree sequence. Hence the two groups have the same number of elements of the same order. It is known that two abelian groups are isomorphic if and only if they have the same number of elements of the same order. That leads to our assertion. □

We also have:

**Proposition 3.** Let \(\Gamma\) be a connected bipartite and regular \(G\)-graph of degree \(p\), \(p\) being a prime number, then either \(\Gamma\) is simple or \(\Gamma\) is of order \(2\).

**Proof.** The graph \(\Gamma\) is bipartite and regular of degree \(p\), so \(\Gamma = \Phi(G, \{s_1, s_2\})\) with \(s_1\) and \(s_2\) two different elements of order \(p\). But \(\Gamma\) is a connected graph, so the family \(\{s_1, s_2\}\) generates the group \(G\), in other words \(G = \langle s_1, s_2 \rangle\). We can notice that the groups \(\langle s_1 \rangle\) and \(\langle s_2 \rangle\) are isomorphic to the cyclic group of order \(p\) called \(C_p\). If \(\langle s_1 \rangle\) and \(\langle s_2 \rangle\) are not different we have:
\[ \langle s_1 \rangle = \langle s_2 \rangle = \langle s_1, s_2 \rangle = G \]

Therefore \(\Gamma\) is the graph of the cyclic group \(C_p\) generated by a family \(S\), with \(S\) containing two elements of order \(p\), so the order of the graph \(\Gamma\) is \(2\). Now let us consider the case where \(\langle s_1 \rangle\) and \(\langle s_2 \rangle\) are different. It is equivalent to saying that for all \(t \in \{1, 2, \ldots, p-1\}\), for all \(k \in \{1, 2, \ldots, p-1\}\) we have:
\[ s_1^t \neq s_2^t, \]

because if \(s_1^t = s_2^t\), \(p\) being a prime number, \(s_2^t\) is generator of \(\langle s_1 \rangle\), and the following equality becomes true:
\[ \langle s_1 \rangle = \langle s_2 \rangle = \langle s_2 \rangle = \langle s_1, s_2 \rangle = G \]

Consequently the only edge between \((s_1)x\) and \((s_2)y\) is the edge corresponding to \(\epsilon\). More generally, let \((s_1)x\) and \((s_2)y\) be two cycles. If \(x = y\), let us suppose that there exist \(t \in \{1, 2, \ldots, p-1\}\), and \(k \in \{1, 2, \ldots, p-1\}\) such that \(s_1^t x = s_2^k y\). We have \(s_1^t = s_2^k\), and that led us to the first case. So there can be only one edge between \((s_1)x\) and \((s_2)y\): the edge corresponding to the element \(x\). Let us consider the case where \(x\) and \(y\) are different. If there is a multi-edge between \((s_1)x\) and \((s_2)y\), then \(s_1^t x = s_2^m y\) and \(s_1^t x = s_2^k y\).

We can suppose that \(t = t + n\) and \(i = k + m\). So we have the following equalities:
\[ s_1^t x = s_1^{t+n} x = s_1^n (s_1^t x) \]
\[ s_1^t x = s_2^m y = s_2^m (s_2^k y) \]

So \(s_1^n (s_1^t x) = s_2^m (s_2^k y)\), but \(s_1^n x = s_2^m y\), consequently \(s_1^n = s_2^m\), and that led us to the first case. □

We will use this well-known result:

**Theorem 2.** [14]
Let \(\Gamma\) be a simple graph. Then \(\text{Aut}_1(\Gamma) \simeq \text{Aut}^*(\Gamma)\) if and only if

(a) not both \(G_1\) and \(G_2\), are components of \(\Gamma\)

(b) and none of the graphs \(G_i, i \in \{3, 4, 5\}\), is a component of \(\Gamma\).

We are now in position to characterize the bipartite \(G\)-graphs:

**Theorem 3.** Let \(\Gamma = (V_1, V_2; E)\) be a bipartite connected semi-regular simple graph. Let \((G, \{s_1, s_2\})\) be a group with \(x = x \in V_1\) and \(y = y \in V_2\). The three following properties are equivalent:

(i) The graph \(\Gamma = (V_1, V_2; E)\) is a \(G\)-graph, \(\Phi(G, \{s_1, s_2\})\).

(ii) the line graph \(L(\Gamma)\) is a Cayley graph \(\text{Cay}(H; \{A\})\), where \(A = \langle a_1 \rangle \cup \langle a_2 \rangle \setminus \{e\}\) with \((G, \{s_1, s_2\}) \simeq (H, \{a_1, a_2\})\)

(iii) the group \(G\) is a subgroup of \(\text{Aut}^*(\Gamma)\) which acts regularly on the set of edges \(E\).
5. THE CONSTRUCTION OF SYMMETRIC
AND SEMISYMMETRIC GRAPHS

5.1 Quartic \( \Gamma \)-graphs

Let \( G \) be a group of order \( 4n \), \( G \neq C_4 \), and \( S \) a family such that \( G = \langle S \rangle \) and \( S = \{a, b\} \), with \( a^4 = e \) and \( b^4 = e \). Then the graph \( \Phi(G; S) \) is bipartite, edge-transitive and quartic. So there are two possibilities:

1. \( \Phi(G; S) \) is vertex-transitive, so it is a symmetric quartic graph;
2. \( \Phi(G; S) \) is not vertex-transitive, so it is a semisymmetric quartic graph.

Therefore \( G \)-graphs are a very interesting tool for constructing quartic edge-transitive graphs, especially semisymmetric graphs. We establish a list of all small groups \( G \) of order \( 4n \) such that \( G \) is generated by two elements of order 4. For that we use GAP and the SmallGroups library. This library gives access to all groups of certain small orders. The groups are sorted by their orders and they are listed up to isomorphism. For computing the list of the groups generated by two elements of order 4 we use the following algorithm:

\[
\text{result} = [];
\text{for all } g \text{, group of order } n \text{ order4=1;} \\
\text{if order}(x) = 4 \text{ then} \\
\text{add } x \text{ to order4} \\
\text{end if} \\
\text{end for} \\
\text{for all } x1 \text{ in order4} \\
\text{for all } x2 \text{ in order4, } x2 \neq x1 \\
\text{if } x1, x2 \rightarrow g \text{ add } (g, x1, x2) \text{ to result} \\
\text{end for} \\
\text{end for} \\
\text{return result;} \\
\]

After the list is established, it is easy to generate all the corresponding \( G \)-graphs and to compute their automorphism group with Nauty [17]. If there is only one orbit in the vertex automorphism group, then the graph is vertex-transitive and symmetric. Otherwise, the graph is semisymmetric and there are two orbits, because there is a theorem [14] which affirms that every semisymmetric graph is bipartite. With that algorithm we establish a list of almost all quartic symmetric graphs up to the order 126. The following tables are not exhaustive. When two or more groups give isomorphic quartic graph the name of the two groups are given in the column \( G \). For more information the full tables are on-line at: http://users.info.unicaen.fr/~bretto (in Publications).
It is a symmetric quartic graph isomorphic to the cuboctahedral graph. It is also a $G$-graph generated by the group $G = C_2 \times C_2 \times C_2$.

5.2 Cubic and quintic symmetric or semisymmetric graphs

By the same process it is easy to establish some tables of all quintic and cubic symmetric and semisymmetric $G$-graphs. Such tables can be found on-line at:


These two table have been built in 38 minutes on a 2 gigahertz Athlon (a 32 bit processor).

By the implication $(i) \Rightarrow (ii)$ of Theorem 3, the linegraph of a cubic $G$-graph is a quartic CAYLEY graph. Our table of cubic symmetric and semisymmetric $G$-graphs gives us a table of quartic CAYLEY graph.

Example:
Let $G$ be the group $sg-12-3$ generated by a family $S = \{a, b\}$ of order 2 with $a^3 = b^3 = e$. For information the group named $sg-12-3$ is isomorphic to $A_4$. The graph $\Phi(G; S)$ is the following:

![Quartic semisymmetric simple G-graphs](https://example.com/quatircle_graph.png)

It is a cubic symmetric $G$-graph on 8 vertices isomorphic to the skeleton of a cube. If we compute the linegraph of the $G$-graph $\Phi(G; S)$ we found the following quartic CAYLEY graph:

```
| $O(\Gamma)$ | $O(G)$ | $G$ | $|Aut(\Gamma)|$ |
|------------|--------|-----|----------------|
| 24         | 48     | $sg-48-30$ | 3072 |
| 32         | 64     | $sg-64-9, sg-64-20$ | 294912 |
| 48         | 96     | $sg-96-194, sg-96-195$ | 3145728 |
| 64         | 128    | $sg-128-122, sg-128-134$ | 25165824 |
| 64         | 128    | $sg-128-144, sg-128-146$ | 2048 |
| 72         | 144    | $sg-144-115, sg-144-116$ | 37748736 |
| 72         | 144    | $sg-144-120$ | 144 |
| 80         | 160    | $sg-160-83, sg-160-84$ | 83886080 |
| 80         | 160    | $sg-160-234$ | 335544320 |
| 96         | 192    | $sg-192-185$ | 192 |
| 96         | 192    | $sg-192-26, sg-192-32, \ldots$ | 1.649e12 |
| 100        | 200    | $sg-200-42$ | 400 |
| 120        | 240    | $sg-240-107, sg-240-192$ | 1.055e15 |
| 120        | 240    | $sg-240-91$ | 1440 |
| 120        | 240    | $sg-240-95, sg-240-97$ | 2.577e11 |
| 120        | 240    | $sg-240-95, sg-240-97$ | 7.31e11 |
```

Notice that the following well-known cubic symmetric or semisymmetric graphs are $G$-graphs. The corresponding groups are indicated between parenthesis:

1. The cube $G = A_4$, $S = \{(1,2,3), (1,3,4)\}$
2. The Heawood’s graph $(a,b | a^7 = b^3 = e, ab = baa), S = \{b, ba\}$
3. The Pappus’s graph $G = (a,b,c | a^3 = b^3 = e, ab = ba, ac = ca, bc = cb), S = \{b, c\}$
4. The Mobius-Kantor’s graph $G =$ SmallGroup$(24,3), S = \{f1, f1 * f2\}$
5. The Gray graph
\((G = \text{SmallGroup}(81,7), S = \{f_1, f_2\})\)

6. The Ljubljana graph
\((G = \text{SmallGroup}(168,43), S = \{f_1, f_1 \ast f_2 \ast f_4\})\)

A census of all symmetric and semisymmetric cubic graphs up to 768 vertices has already been established [8, 9], but our non-exhaustive algorithm is faster. For computing almost all cubic symmetric and semisymmetric G-graphs up to the order 800, except order 768, 1 hour and 56 minutes are necessary on a 2 Gigahertz Athlon.

No list was established for quintic symmetric or semisymmetric graphs. The two following non-exhaustive lists, built in 32 minutes on a 2 gigahertz Athlon, are the first one:

<table>
<thead>
<tr>
<th>Quintic semisymmetric G-graphs</th>
<th>O(1)</th>
<th>O(G)</th>
<th>G</th>
<th>Aut(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>300</td>
<td>sg-300-22</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>600</td>
<td>sg-600-54</td>
<td>2400</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>625</td>
<td>sg-625-7</td>
<td>10000</td>
<td></td>
</tr>
<tr>
<td>720</td>
<td>1800</td>
<td>sg-1800-555</td>
<td>14400</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quintic symmetric G-graphs</th>
<th>O(1)</th>
<th>O(G)</th>
<th>G</th>
<th>Aut(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>25</td>
<td>sg-25-2</td>
<td>28800</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>55</td>
<td>sg-55-1</td>
<td>1320</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>60</td>
<td>sg-60-5</td>
<td>480</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>80</td>
<td>sg-80-19</td>
<td>3840</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>120</td>
<td>sg-120-5</td>
<td>960</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>125</td>
<td>sg-125-3</td>
<td>4000</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>155</td>
<td>sg-155-1</td>
<td>310</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>160</td>
<td>sg-160-199</td>
<td>640</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>205</td>
<td>sg-205-1</td>
<td>410</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>275</td>
<td>sg-275-3</td>
<td>550</td>
<td></td>
</tr>
<tr>
<td>122</td>
<td>305</td>
<td>sg-305-1</td>
<td>610</td>
<td></td>
</tr>
<tr>
<td>128</td>
<td>320</td>
<td>sg-320-1012</td>
<td>2560</td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>355</td>
<td>sg-355-4</td>
<td>710</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>360</td>
<td>sg-360-118</td>
<td>5760</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>400</td>
<td>sg-400-213</td>
<td>3200</td>
<td></td>
</tr>
<tr>
<td>162</td>
<td>405</td>
<td>sg-405-15</td>
<td>19440</td>
<td></td>
</tr>
<tr>
<td>202</td>
<td>505</td>
<td>sg-505-1</td>
<td>1010</td>
<td></td>
</tr>
<tr>
<td>242</td>
<td>605</td>
<td>sg-605-5</td>
<td>1210</td>
<td></td>
</tr>
<tr>
<td>242</td>
<td>605</td>
<td>sg-605-6</td>
<td>1210</td>
<td></td>
</tr>
<tr>
<td>242</td>
<td>605</td>
<td>sg-605-6</td>
<td>2420</td>
<td></td>
</tr>
<tr>
<td>262</td>
<td>655</td>
<td>sg-655-1</td>
<td>1310</td>
<td></td>
</tr>
<tr>
<td>264</td>
<td>660</td>
<td>sg-660-13</td>
<td>5280</td>
<td></td>
</tr>
<tr>
<td>288</td>
<td>720</td>
<td>sg-720-409</td>
<td>2880</td>
<td></td>
</tr>
<tr>
<td>302</td>
<td>755</td>
<td>sg-755-3</td>
<td>1510</td>
<td></td>
</tr>
<tr>
<td>310</td>
<td>775</td>
<td>sg-775-3</td>
<td>1550</td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>800</td>
<td>sg-800-1065</td>
<td>6400</td>
<td></td>
</tr>
<tr>
<td>352</td>
<td>880</td>
<td>sg-880-214</td>
<td>1760</td>
<td></td>
</tr>
<tr>
<td>362</td>
<td>905</td>
<td>sg-905-1</td>
<td>1810</td>
<td></td>
</tr>
<tr>
<td>382</td>
<td>955</td>
<td>sg-955-1</td>
<td>1910</td>
<td></td>
</tr>
<tr>
<td>384</td>
<td>960</td>
<td>sg-960-11357</td>
<td>7680</td>
<td></td>
</tr>
<tr>
<td>384</td>
<td>960</td>
<td>sg-960-11358</td>
<td>40800</td>
<td></td>
</tr>
<tr>
<td>410</td>
<td>1025</td>
<td>sg-1025-5</td>
<td>2850</td>
<td></td>
</tr>
<tr>
<td>452</td>
<td>1080</td>
<td>sg-1080-260</td>
<td>3730</td>
<td></td>
</tr>
<tr>
<td>482</td>
<td>1205</td>
<td>sg-1205-1</td>
<td>2410</td>
<td></td>
</tr>
<tr>
<td>486</td>
<td>1215</td>
<td>sg-1215-58</td>
<td>4860</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group ID</th>
<th>Group ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>502</td>
<td>sg-1255-1</td>
</tr>
<tr>
<td>528</td>
<td>sg-1320-13</td>
</tr>
<tr>
<td>542</td>
<td>sg-1355-1</td>
</tr>
<tr>
<td>350</td>
<td>sg-1375-9</td>
</tr>
<tr>
<td>562</td>
<td>sg-1405-1</td>
</tr>
<tr>
<td>680</td>
<td>sg-1500-112</td>
</tr>
<tr>
<td>610</td>
<td>sg-1525-5</td>
</tr>
<tr>
<td>622</td>
<td>sg-1555-1</td>
</tr>
<tr>
<td>640</td>
<td>sg-1600-6786</td>
</tr>
<tr>
<td>662</td>
<td>sg-1655-1</td>
</tr>
<tr>
<td>682</td>
<td>sg-1705-3</td>
</tr>
<tr>
<td>682</td>
<td>sg-1705-4</td>
</tr>
<tr>
<td>682</td>
<td>sg-1705-5</td>
</tr>
<tr>
<td>682</td>
<td>sg-1705-6</td>
</tr>
<tr>
<td>704</td>
<td>sg-1760-1139</td>
</tr>
<tr>
<td>710</td>
<td>sg-1775-3</td>
</tr>
<tr>
<td>722</td>
<td>sg-1805-2</td>
</tr>
<tr>
<td>800</td>
<td>sg-2000-488</td>
</tr>
</tbody>
</table>

6. REFERENCES


