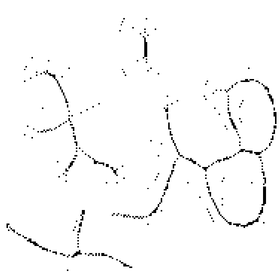
# FROM NON CONNECTED TO HOMOTOPIC SKELETONS IN MULTIDIMENSIONAL DIGITAL SPACES

**Purpose:**

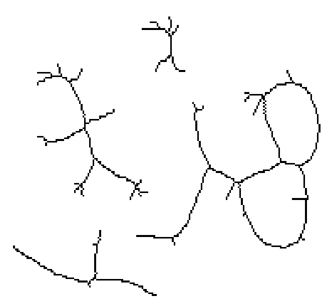
To bridge the gap between the Maximal Balls Skeleton (MBS) and skeletons by thinnings.

**Maximal Balls Skeleton**



- # True skeleton
- # Can be expressed in terms of morphological transforms
- # Reconstruction and shape descriptor capabilities
- # Non connected

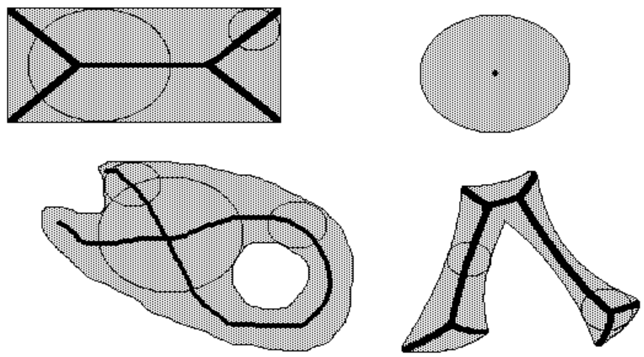
**Skeletons by thinnings**



- # Many algorithms (L,M,D thinnings)
- # No obvious links with the true skeleton
- # Use sequential thinnings (bias)
- # Connected (homotopic transforms)

# Maximal Balls non connected Skeleton

## Maximal Balls



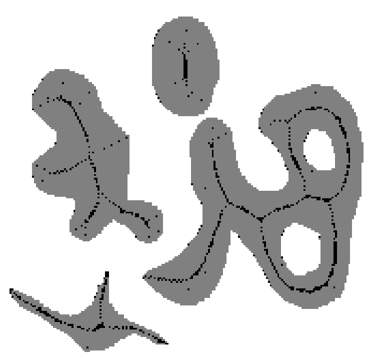
**Skeleton of a set X** : Set of all the centers of the maximal balls of X.

## Skeleton and Morphological Openings

$$S(X) = \bigcup_{i=0}^{\infty}[(X \ominus iB)/(X \ominus iB)_B]$$

**Lantuéjoul's Formula**

The Maximal Balls Skeleton S(X) is made of the residues by opening of the successive erosions o



This formula does not depend on the dimension of space.

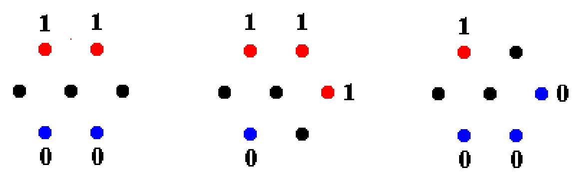
# Connected skeletons using Sequential Thinnings

## Sequential thinnings

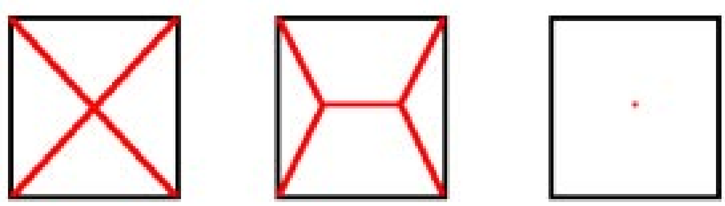
$$(((X \circ T_1) \circ T_2) \circ \ldots\ldots \circ T_n)$$

$T_i$ , successive rotations of an homotopic structuring element.

## Various thinnings produce various skeletons

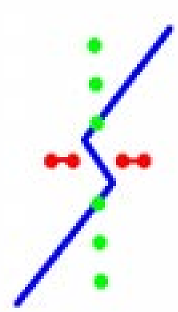



L M D



L- Skeleton M-Skeleton D-Skeleton

## Bias due to rotations



# Skeletons and Thinnings (1)

## Question 1

Is it possible to express the Maximal Balls Skeleton with non sequential thinnings?

$$S(X) = ((X \circ T) \circ T) \circ \ldots$$

$X \circ T$ : **Union thinning** (thinning using a family $T$ of structuring elements)

$$T = \{ T_1, T_2, \ldots , T_i, \ldots \}$$

$$X \circ T = \bigcap_i (X \circ T_i)$$

## Question 2

How to exhibit the family $T$ ?

## Question 3

How to connect the skeleton by selecting among the family $T$ a sub-family $T'$ preserving homotopy?

# Skeleton and Thinnings (2)

## Answer 1

Define an iterative transform:

$$X = Z_0$$

$$Z_n = (Z_{n-1} \ominus B) \cup (Z_{n-1} / (Z_{n-1})_B)$$

One can show that :

$$Z_n = (X \ominus nB) \cup S_{n-1}(X)$$

with :

$$S_{n-1}(X) = \bigcup_{i=0}^{n-1} ((X \ominus iB) / (X \ominus iB)_B)$$

$Z_\infty = S(X)$ , Maximal Balls Skeleton

The general form of this iterative thinning is:

$$(Z \ominus B) \cup (Z / Z_B)$$

$$Z \cap [(Z \ominus B)^c \cap Z_B]^c$$

Hit-or-Miss transform:

$$\bigcup_{a,b \in B} [(Z \ominus B_a) \cap (Z^c \ominus L_b)]$$

$B_a$ , translated ball in direction a
$L_b$ , translated point in direction b

The Maximal Balls Skeleton can be written as a succession of union thinnings with:

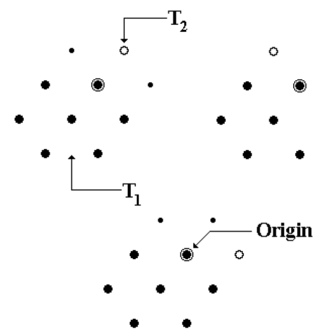
$$T = \{T_{a,b} = (B_a , L_b) \; a,b \in \text{ elementary ball}\}$$

**Examples of T families**

<u>Answer 2</u>

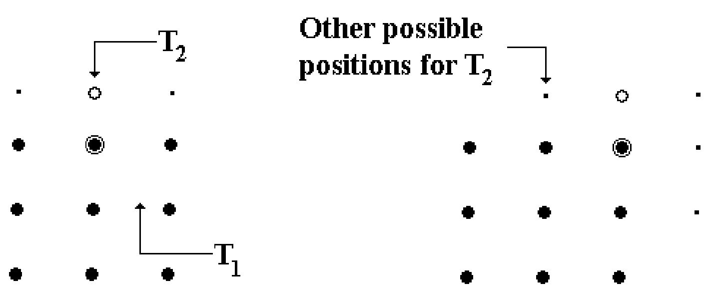
### 2D hexagonal grid

Elementary ball : Hexagon



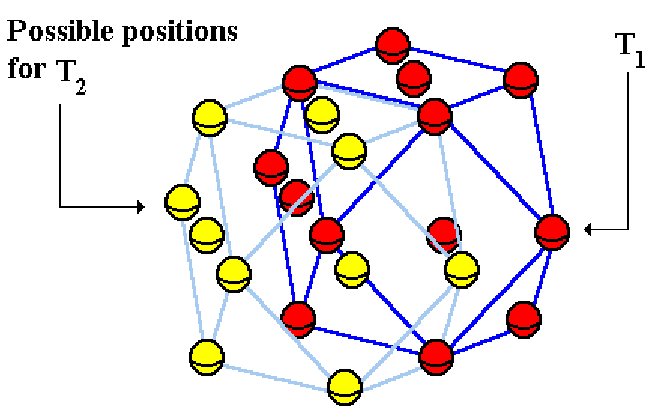

### 2D square grid

Elementary ball : Square




### 3D cubic grid

Elementary ball : Cuboctahedron

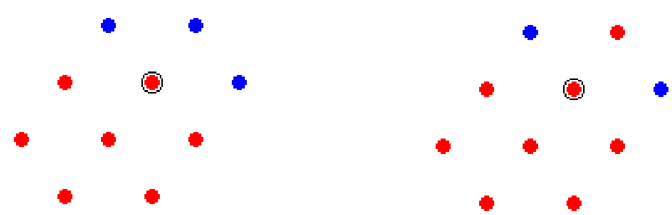


**How to connect the previous skeleton?**

2 possible approaches

<u>Answer 3</u>

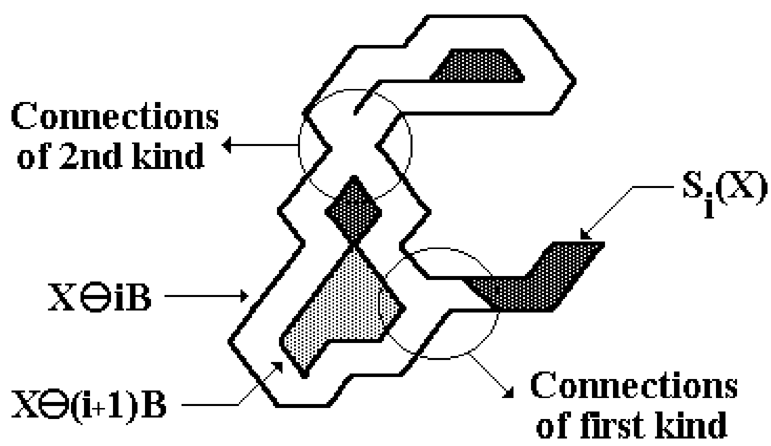
# Use T and sort those structuring elements preserving homotopy



**Homotopy preserving** **Homotopy breaking**

**Main drawback:** complex (especially in 3D) and long procedure.

# Starting from the Maximal Balls Skeleton, define a connection procedure for the residues




**Two kinds of connections:**

- Connection of the residues at step i with the eroded set of size i+1.
- Connection of the connected components of the eroded set of size i+1, when disconnections happen between step i and i+1.

## Connected Skeleton by Union Thinnings

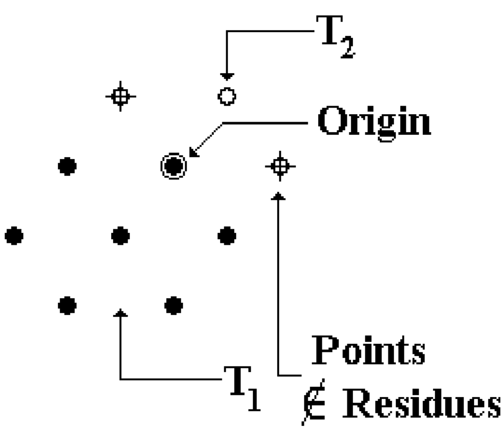
**Sub-family T ' ⊂ T of structuring elements preserving homotopy**

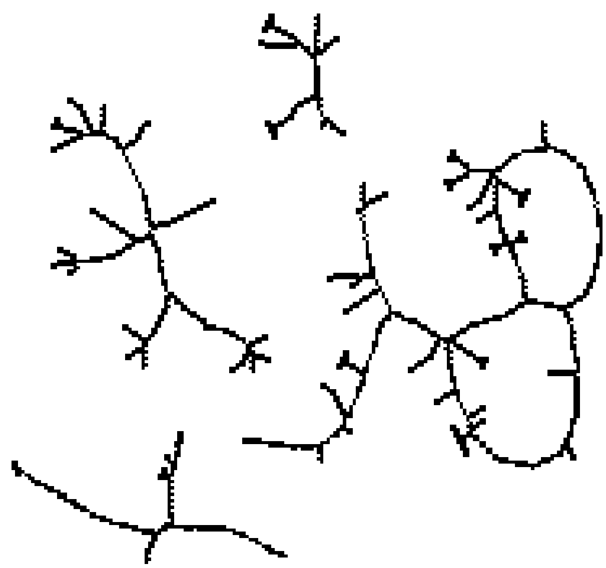
((X o T ') o T ') o ... o T ') o .... = $S_c(X)$

**T '** is made of 3-phases structuring elements:

- Points belonging to $X_i$
- Points belonging to $X_i^c$
- Points not belonging to the residues of $X_i$

**2D example on the hexagonal grid**



**Properties of $S_c$**

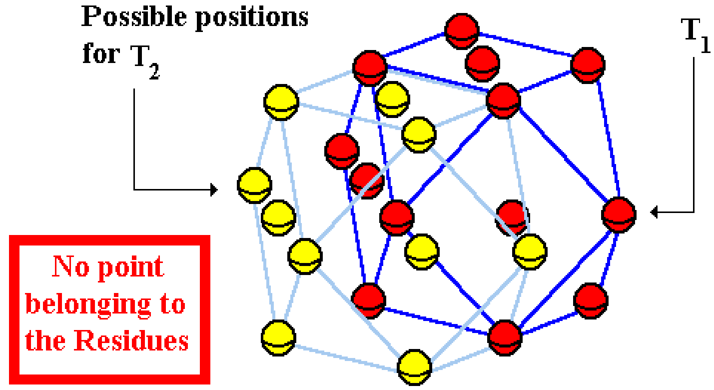
- $S(X) \not\subset S_c(X)$ That means that the minimal set of structuring elements preserving homotopy is not able to produce a "true" connected skeleton.
- If X connected set, then $S(X) \cup S_c(X)$ is connected. Some points of the MBS may have been forgotten by the homotopy thinning, nevertheless these points are adjacent to the connected skeleton.

# 3D Skeleton

**3D connected skeleton on the cubic grid**

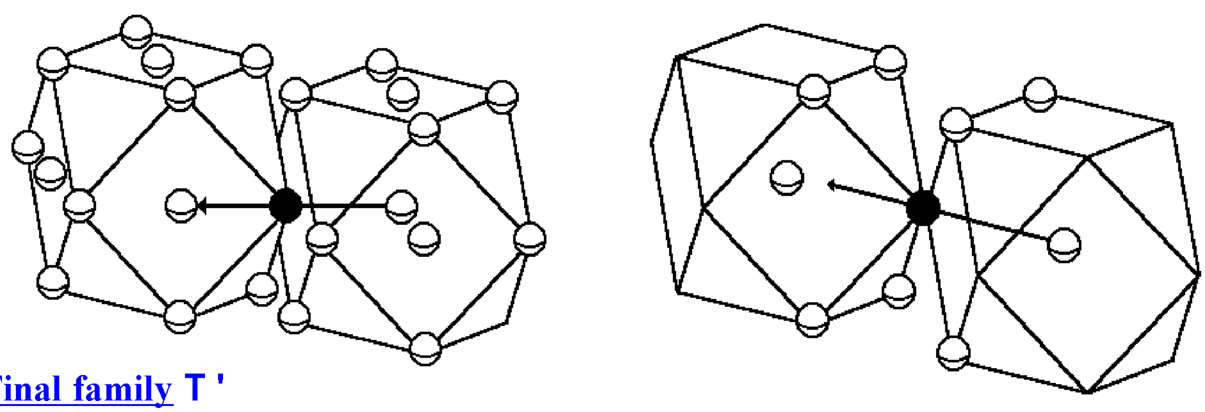
**Elementary ball: cuboctahedron**

\# **Preserving connections of the first kind**

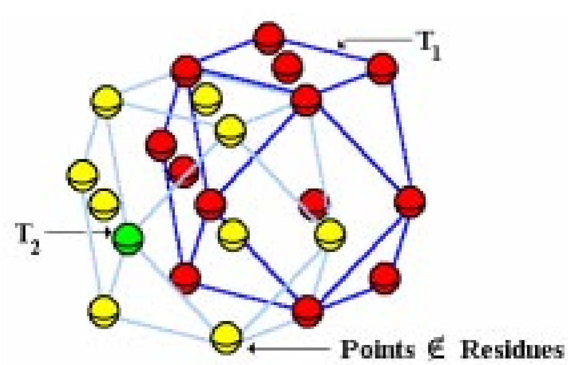



\# **Preserving connections of second kind**

Examples of second kind connections



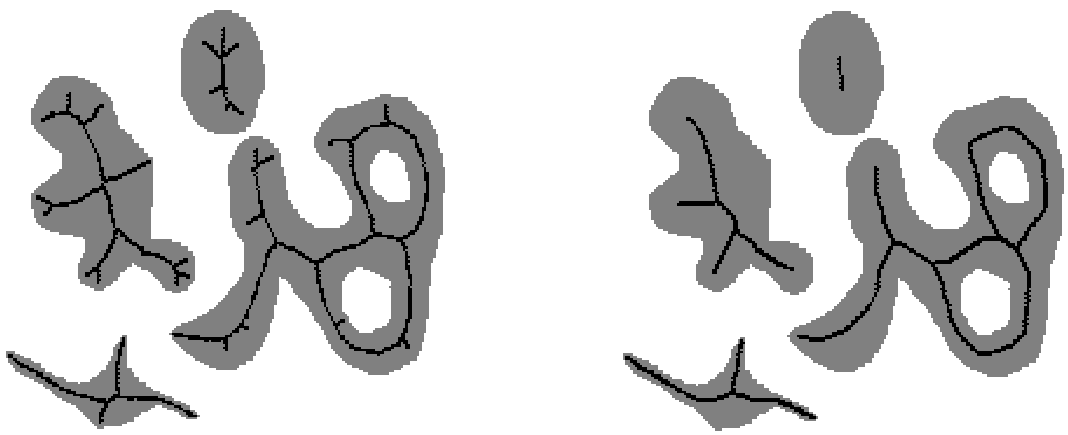
**Final family** T '



## Other applications

# **Smooth skeletons**

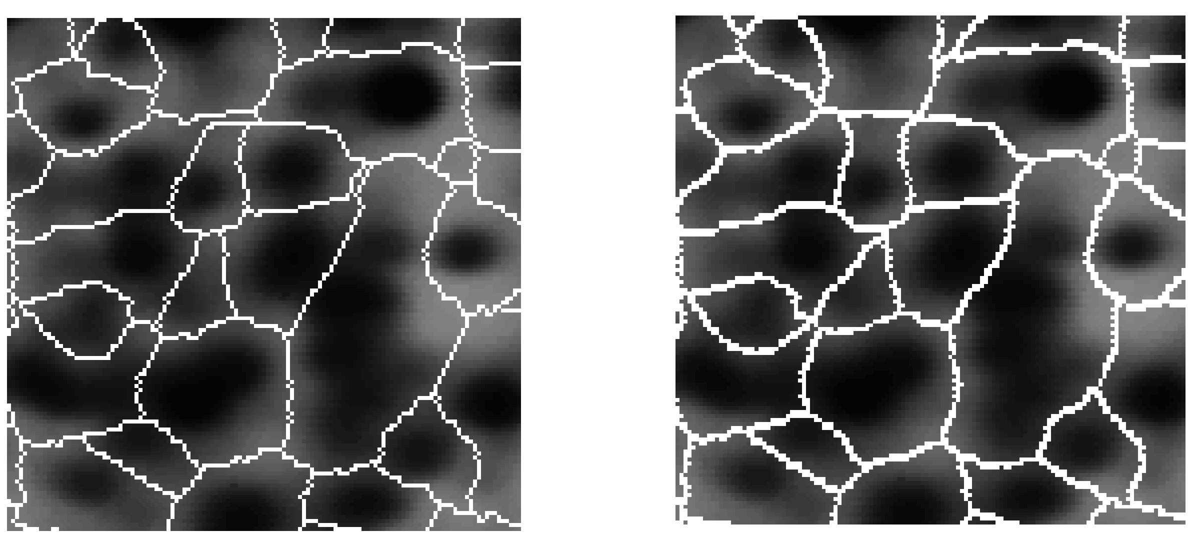
$T' \longrightarrow T''$ , Reduced homotopy preserving family



Smooth skeleton when X, regular set ($X_B$=X)

# **Geodesic skeletons**

# **Skeletons for functions, watersheds**



Comparison between a watershed using thinnings with M structuring elements and a watershed performed with the $T'$ family.