

# How to simulate a volume-controlled flooding with mathematical morphology operators?

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**Note:** To follow this note and especially to get definitions of concepts as lower catchment basin, FOZ, dual geodesic reconstruction, I STRONGLY recommend to read the following documents available on my Web page (<http://cmm.ensmp.fr/~beucher/publi.html>):

- P algorithm, a dramatic enhancement of the waterfal transformation ([http://cmm.ensmp.fr/~beucher/publi/P-Algorithm\\_SB\\_BM.pdf](http://cmm.ensmp.fr/~beucher/publi/P-Algorithm_SB_BM.pdf), in particular, pages 1 to 10) [1]
- Watersheds and waterfalls ([http://cmm.ensmp.fr/~beucher/slideshow/cours2000\\_fichiers/frame.htm](http://cmm.ensmp.fr/~beucher/slideshow/cours2000_fichiers/frame.htm)) [2]

## The problem

The classical tool for flooding simulation is the watershed transform. However, the flooding is simply controlled by its height. This means that, when the flooding has reached an altitude where it can flood an adjacent catchment basin (Fig 1), this catchment basin and also other catchment basins are immediately totally flooded up to the current level of water. In fact, the topography of these catchment basins has no importance since the flooding is immediate and not controlled by these catchment basins (Fig 2).

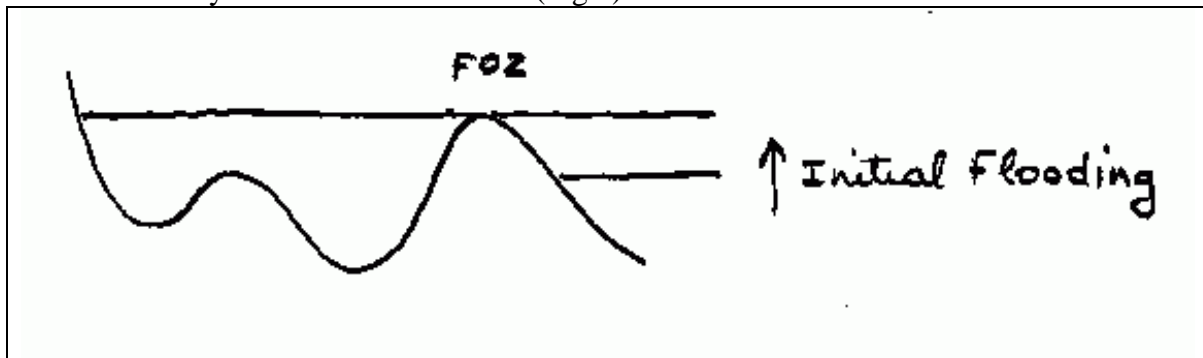


Fig 1: Flooding of the next CB is made immediately and totally. Several catchment basins may be flooded at the same time.

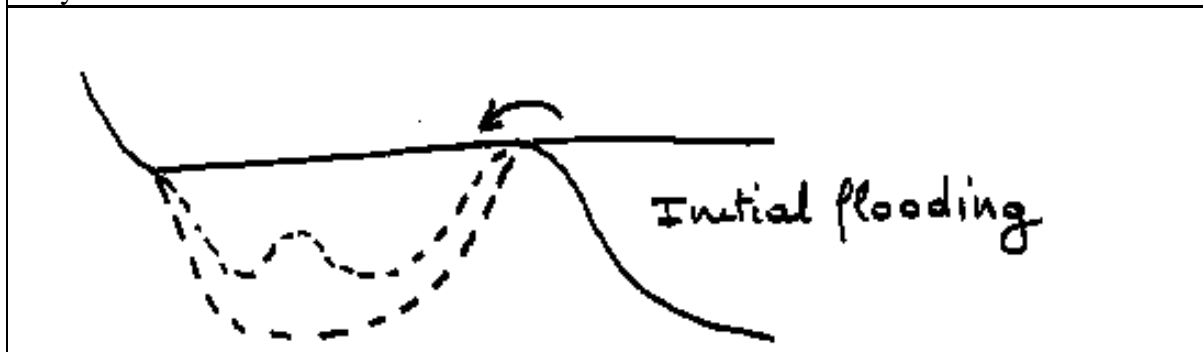


Fig2: The topography of the flooded catchment basins is not taken into account.

Simulating a volume-controlled flooding is not so simple as this process mixes two phenomena: a flooding phenomenon (identical to the flooding process occurring in the watershed transform) but also an overflow which occurs when the flood reaches the first overflow zone (FOZ) of the current flooded catchment basin. This is illustrated at Fig 3.

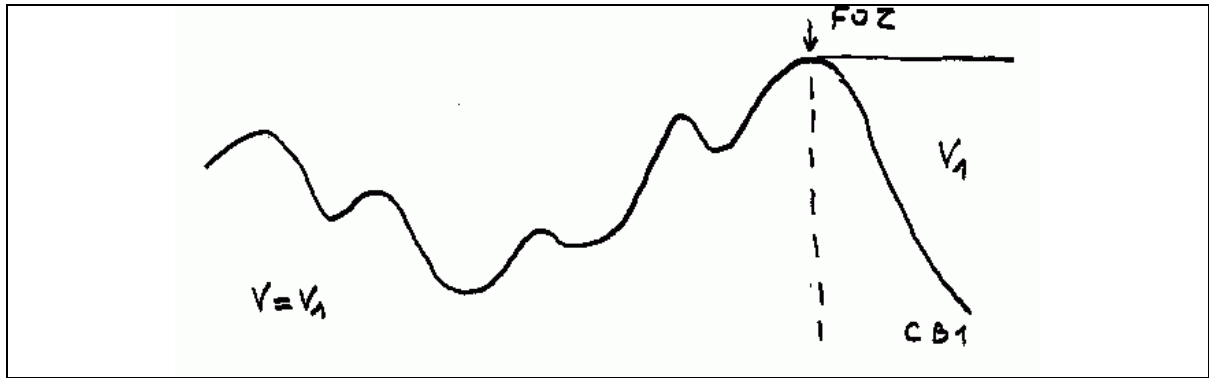


Fig 3-a: First step of flooding

At the first step, a volume  $V_1$  of water is used to flood the initial catchment basin  $CB_1$  up to its first FOZ (Fig 3-a). The flooded region corresponds to the lower catchment basin  $LCB_1$  (see Fig 4 for a 2D illustration). Then, water pours through the FOZ and fills the adjacent catchment basin  $CB_2$  or, more precisely, the corresponding lower catchment basin  $LCB_2$ . In this case, the phenomenon which occurs is a waterfall rather than a flooding. At the end of this step, the volume of water which has been consumed is  $V_1+V_2$ . (Fig 3-b).

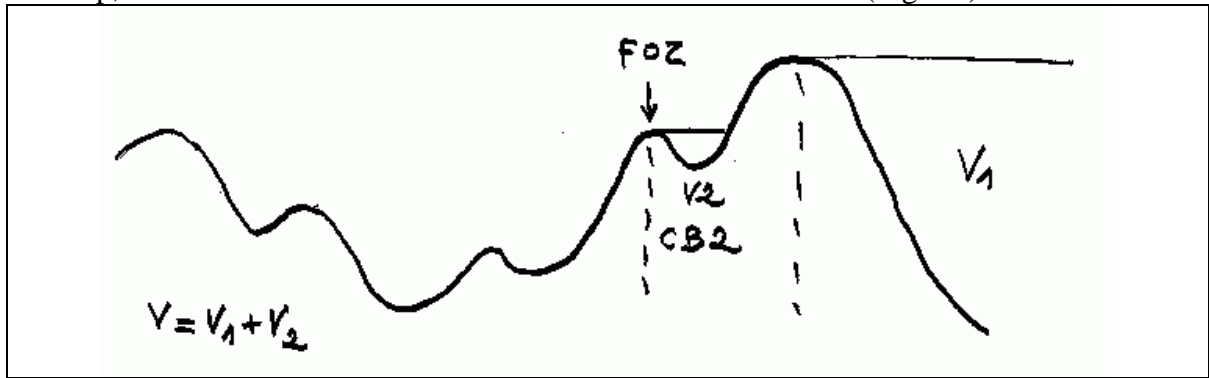


Fig 3-b: Flooding and flooded regions after step 2.

Then, the process goes on and at the third step, we obtain the result given at Fig 3-c. At the next step, the result of the flooding/waterfall process is illustrated at fig 3-d. The catchment basin  $CB_4$  is flooded from the previous one. However, the FOZ of this catchment basin sends the flooding to the previous catchment basin  $CB_3$  which is already flooded! Therefore, the flooding process must be continued in order to reach the FOZ connecting the already flooded regions and the first catchment basin not flooded yet (Fig 3-e).

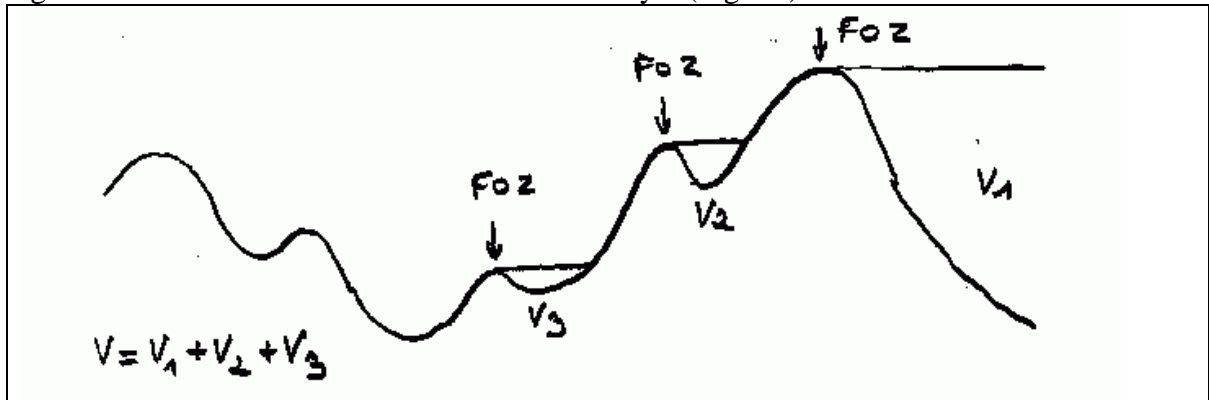


Fig 3-c: flooding after third step.

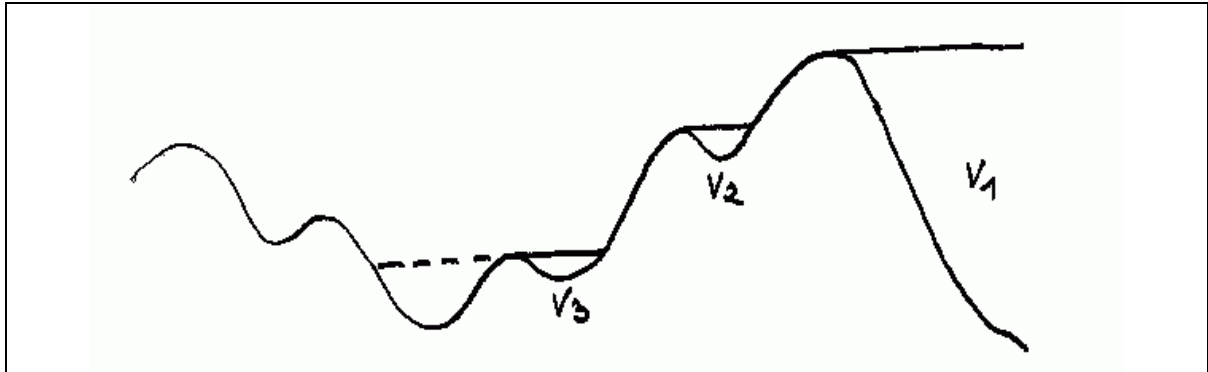


Fig 3-d: Fourth step of flooding, intermediary result. If the next catchment basin is flooded up to its FOZ, the process is looping.

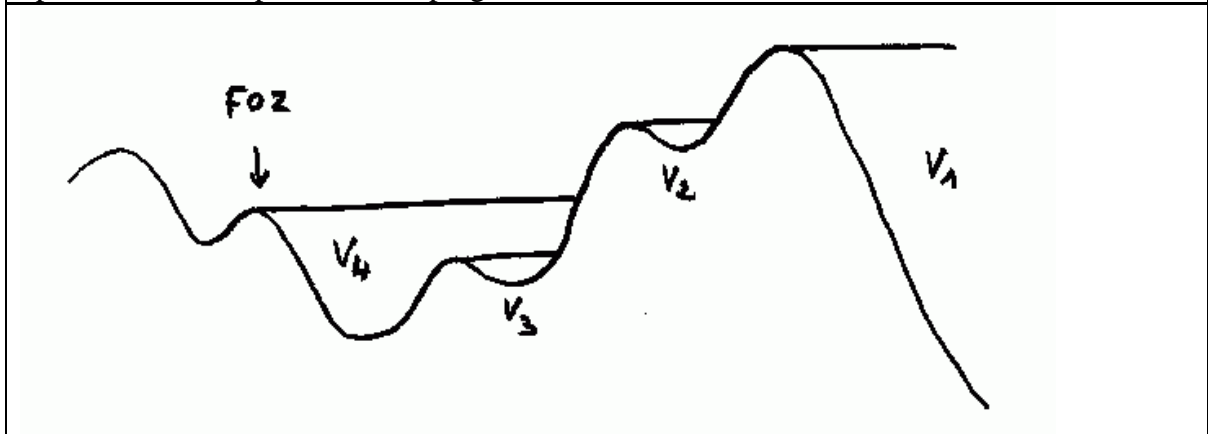


Fig 3-e: Fourth step, final result. The flooding process is continued until the next available FOZ is reached.

The next step is given at Fig 3-f.

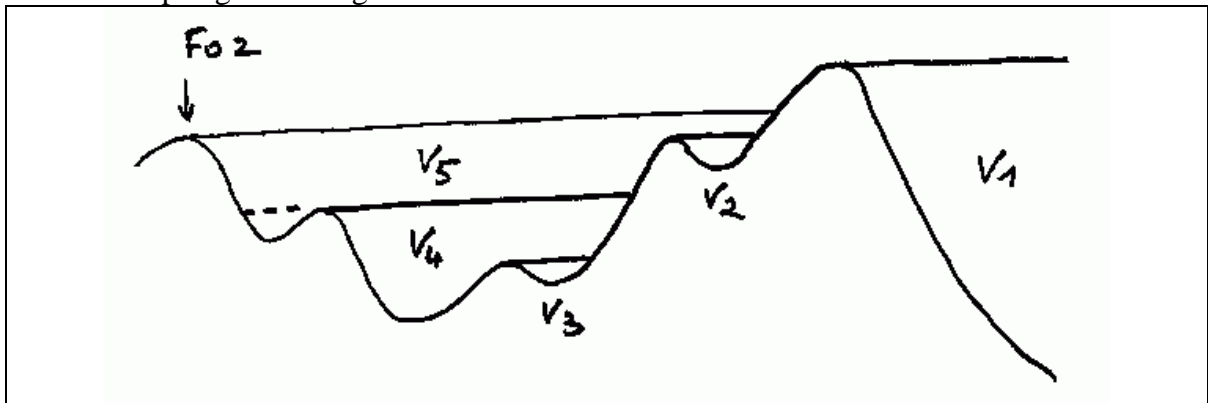


Fig 3-f: Next step of flooding. Here again, the process must be continued after the intermediary flooding as no new FOZ is available.

Note that, when more than one catchment basin is flooded from a previous one, each new catchment basin is filled up to its own FOZ. This means that this procedure establishes, de facto, a flooding order or a hierarchy between the different catchment basins. The flooding of all the catchment basins adjacent to the previously flooded ones is synchronised. These catchment basins will not contribute to the next step of flooding until their corresponding lower catchment basins are all entirely flooded. We shall come back later to the consequences of this particular flooding scheme.

### A flooding algorithm

From the previous analysis, a volume-controlled flooding can be defined. Each step will be illustrated. We suppose that we have the initial valued watershed  $w$  of the DEM  $f_0$ , the binary image of all the catchment basins  $CB$  (which corresponds to the threshold at 0 of the valued watershed  $w$ ). We also suppose, for the sake of simplicity, that the flooding source is unique and embedded in a single catchment basin (although there is no particular problem with this algorithm if this condition is not fulfilled).

At each step of the flooding, two objects will be built:

- the union  $M_i$  of all the catchment basins which have been involved in the flooding at step  $i$  ("involved" means that these catchment basins have been reached by the flood but they are not necessarily totally flooded).  $M_i$  is, by construction, a single connected component (if there is a single source of flooding).
- the flooded DEM  $f_i$  after step  $i$ . The volume of water used at the end of this step corresponds to the volume of  $(f_i - f_0)$ .

The process is initialised by generating  $M_1$ , made of the catchment basin containing the initial source of flooding (Fig 4-a). Then the flooded topography  $f_1$  is built (see below how this can be achieved).  $V_1 = \text{vol}(f_1 - f_0)$  is the volume of water consumed in this first step. The difference of these two functions corresponds to the lower catchment basin  $LCB_1$ . It also corresponds to the lower catchment basin of  $M_1$  (since at this step,  $M_1 = CB_1$ ). Then, the FOZ of this lower catchment basin is determined and the catchment basin adjacent to this FOZ in  $CB$  is added to  $M_1$  to produce  $M_2$ . Note that  $M_2$  must be connected so the part of the watershed line which separates  $M_1$  and the new catchment basin must be entirely removed. We explain later how this is done.

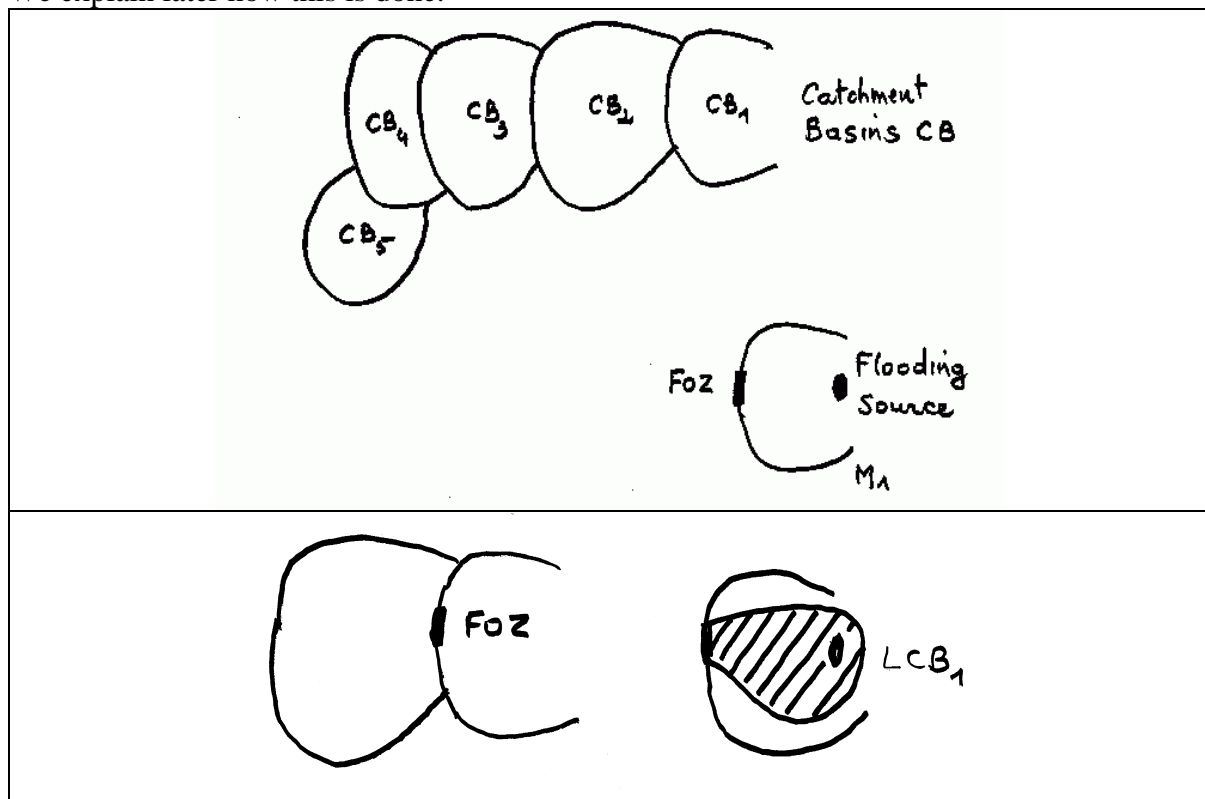


Fig 4-a: 2D representation of the process. Initial catchment basins and generation of  $M_1$  ( $CB_1$ ) from the flooding source. Then, the FOZ is determined by geodesic reconstruction. The corresponding lower catchment basin is built and the next catchment basin is added (it is marked by the FOZ).

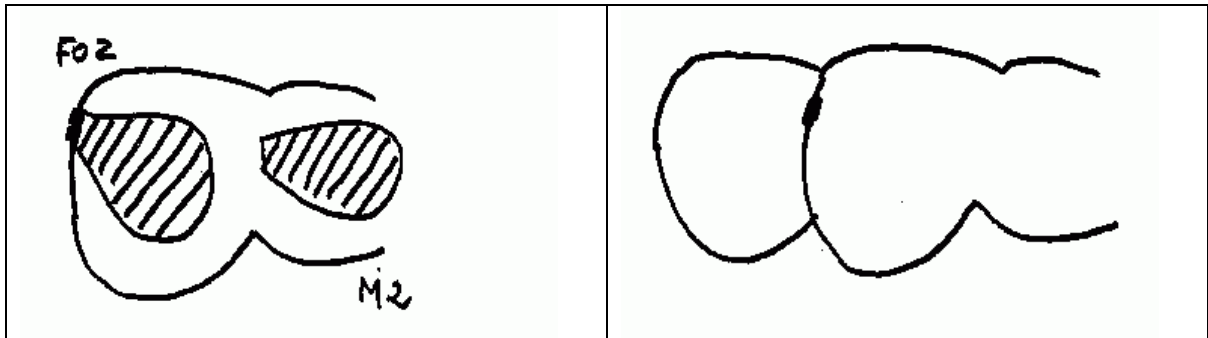


Fig 4-b: Second step. M2 is computed, the flooded corresponding region is determined, the next catchment basin is built from the detected new FOZ

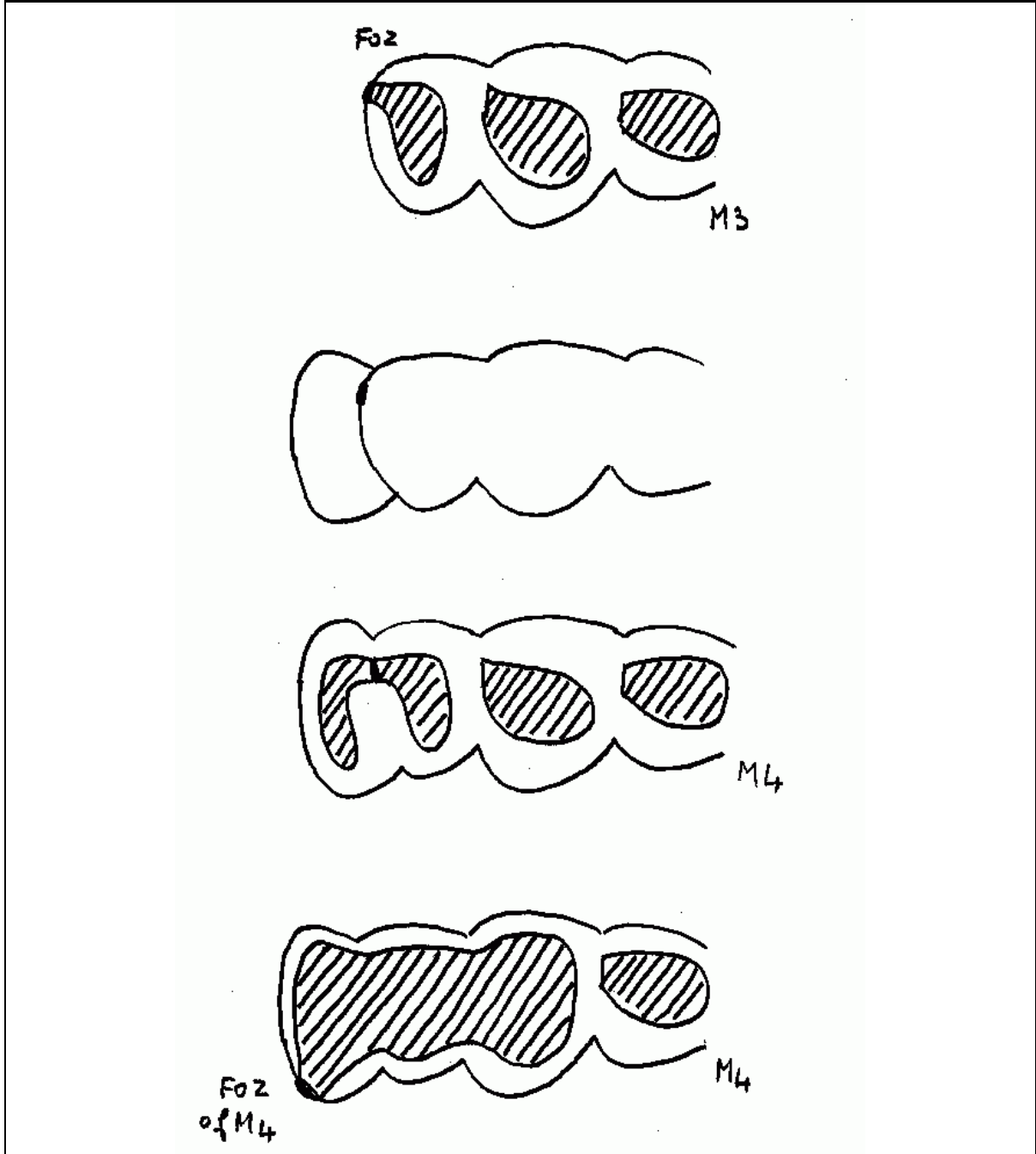


Fig 4-c: Steps 3 and 4 of the process. At step 4, the intermediary result shows that there is no FOZ reached by the flood yet. Therefore, the flooding must be continued up to the FOZ belonging to the boundary of M4.

The next step consists in determining the flooded topography  $f_2$ . The supplementary flooded zone is obviously contained into  $M_2$ . Then, after the determination of the corresponding FOZ, the adjacent catchment basin(s) is (are) added to  $M_2$  to produce  $M_3$ . Fig 4-b and Fig 4-c illustrate these different steps. Note the particular behaviour at step 4. In this case, as we have reached one of the lowest catchment basin, the next flood does not invade a new catchment basin but, on the contrary, it continues to spread into  $M_4$ . This simply means that no waterfall occurs anymore and that the “classical” watershed flooding is acting again.

At each step, the flooded volume can be calculated easily. We can then control this flooding by introducing a maximum available volume of water  $V_{max}$ . If, after step  $i$ , the flooded volume  $V_i$  is lower than  $V_{max}$ , the process goes on. If not, we stop the process at the previous step  $i-1$ . The difference of volume  $V_{max} - V_{i-1}$  can be managed easily since we know which catchment basins should be partly flooded by this quantity of water (see below).

### How to determine $f_i$ and the active FOZ at each step?

Determining  $f_i$  and the current FOZ can be achieved with the geodesic reconstruction. In fact, the procedure is similar to the algorithm which is used to build the hierarchical image (see page of [1] or slide in [2]). The only difference is that this procedure is not applied on the whole valued watershed but only on the part of it which corresponds to the boundary of  $M_i$  at step  $i$ .

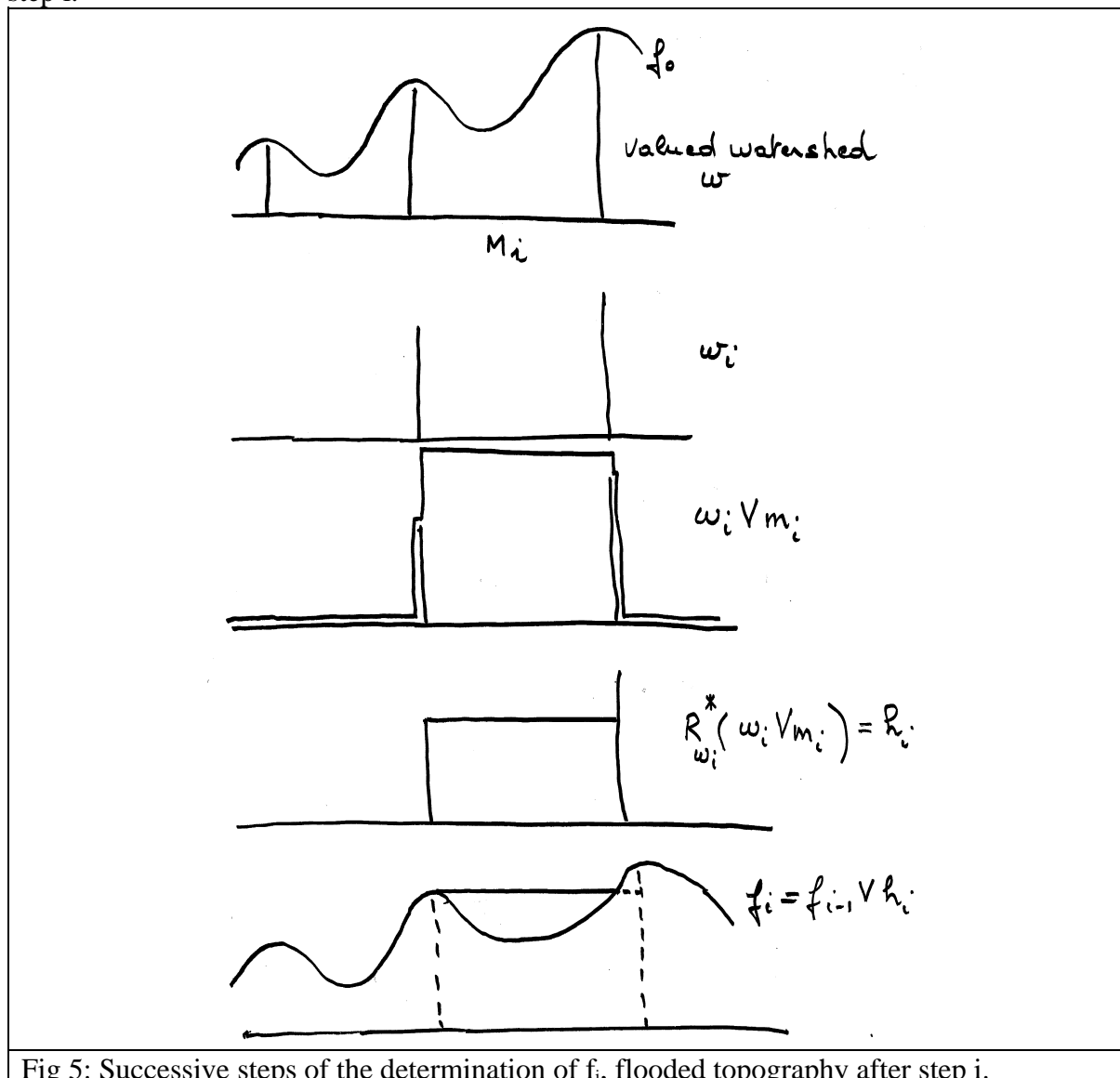


Fig 5: Successive steps of the determination of  $f_i$ , flooded topography after step  $i$ .

Let  $M_i$  be the (partly) flooded region at step  $i$  and  $m_i$  its corresponding valued mask. The operation:

$$w \wedge [\delta(m_i) - m_i] = w_i$$

gives the valued boundary of  $M_i$ .

The dual reconstruction of  $w_i$  with  $w_i \vee m_i$  as marker function gives the “local hierarchical image of  $w_i$  (hierarchical image limited to  $M_i$ ):

$$h_i = R_{w_i}^*(w_i \vee m_i)$$

$f_i$  is then equal to  $f_i = f_{i-1} \vee h_i$  (Fig 5).

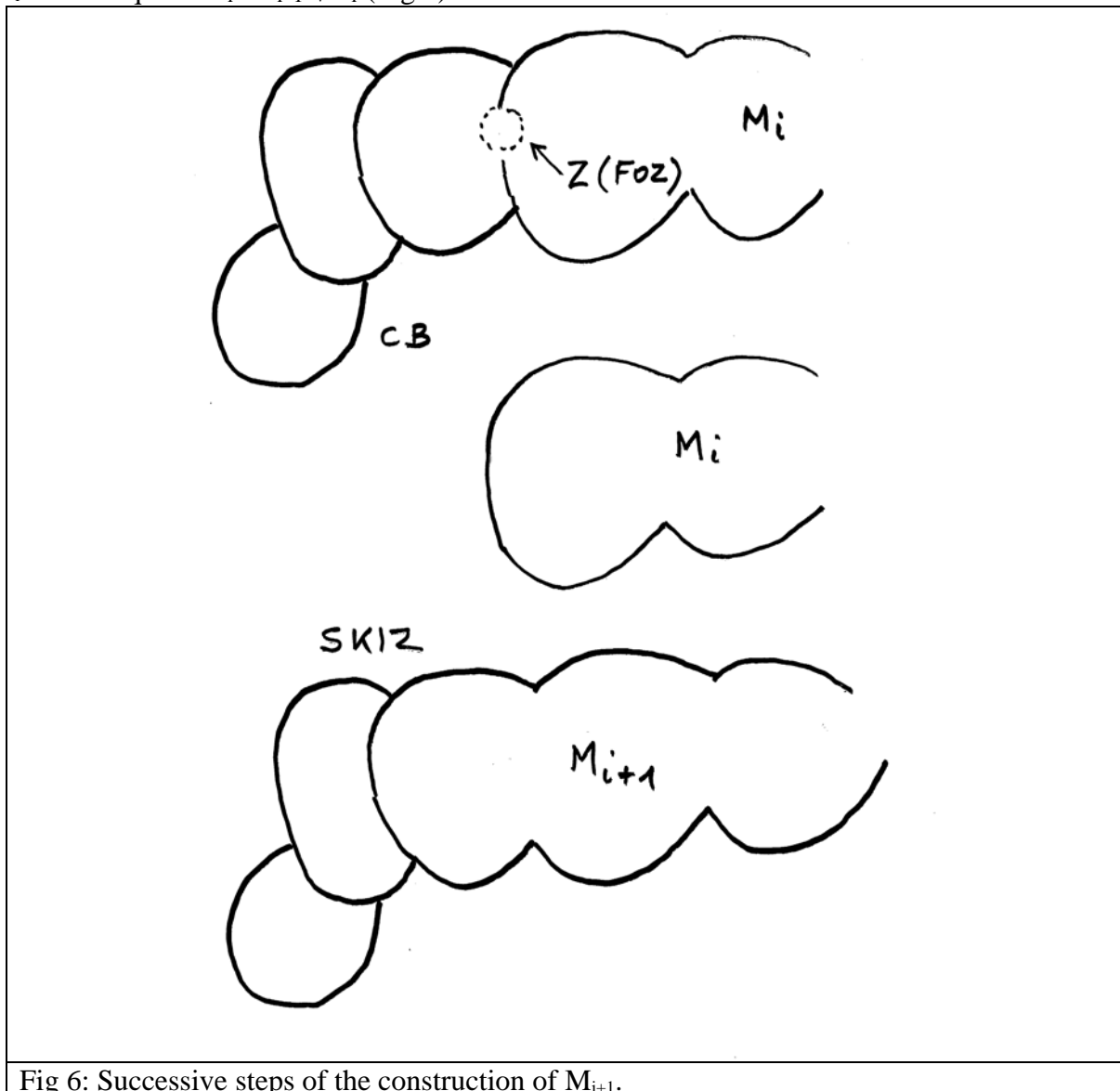


Fig 6: Successive steps of the construction of  $M_{i+1}$ .

Getting the FOZ of  $M_i$  is quite complicated. The procedure described in [1] is a good start but it is not sufficient as some FOZ may be large (FOZ corresponding to triple points of the watershed).

Firstly, we determine the points for which  $\delta(h_i \wedge m_i) = w_i$ . Let  $Z$  be this set and  $z$  the corresponding numerical mask. Then, we must iterate the following operations until idempotence:

- compute  $\delta(w_i \wedge z)$
- generate  $Z = \{\delta(w_i \wedge z) = w_i\}$  and the corresponding numerical mask  $z$ .

The determination of  $M_{i+1}$  is obtained by:

$$M_{i+1} = R[\text{SKIZ}(M_i \cup \text{CB} \cup Z), M_i]$$

Where SKIZ produces the zones of influence of the set  $(M_i \cup \text{CB} \cup Z)$  and R is the geodesic reconstruction (Fig 6).

### Algorithm description

Here are the different steps of this algorithm.

#### Initial data:

- The DEM image  $f_0$
- The initial source of flooding, S (it is a set).
- The maximum volume of water available,  $V_{\max}$ .

#### Initialisation

1. Compute  $w$ , the valued watershed of  $f_0$ .
2. Generate the catchment basins image CB of  $w$  (set obtained by thresholding  $w$  at 0).
3. Compute  $M = \text{build}(\text{CB}, S)$ ;  $M$  is the set of all the (even partly) flooded catchment basins at the end of each step.
4. Generate  $m$  (valued mask of  $M$ , obtained by a convert function).
5. Let the current flooded topography  $f_i$  be equal to  $f_0$ .

#### Process

1.  $f_{i-1} = f_i$  ( $f_{i-1}$  is the flooded topography at the beginning of each step).
2.  $M' = M$  ( $M'$  indicates the partly flooded catchment basins at the beginning of each step).
3. Compute  $m'$  (convert function applied to  $M'$ )
4. Calculate  $w_i = w \wedge [\delta(m') - m']$
5. Calculate  $h_i = R_{w_i}^*(w_i \vee m')$  (dualbuild of  $(w_i \vee m')$  over  $w_i$ ).
6. Calculate  $f_i = f_{i-1} \vee h_i$
7. Calculate  $V = \text{vol}(f_i - f_0)$ , total volume of water consumed since start.
8. Calculate  $V_i = \text{vol}(f_i - f_{i-1})$ , volume used at step  $i$  (optional).
9. Call  $\text{getFOZ}(m', w_i, h_i)$ , result is the FOZ  $Z$  (see description of this function below).
10. Compute  $\text{WRK}_1 = M' \cup \text{CB} \cup Z$
11.  $\text{WRK}_1 = \text{SKIZ}(\text{WRK}_1)$
12.  $M = \text{build}(\text{WRK}_1, M')$ , geodesic reconstruction.
13. If  $(V \leq V_{\max})$ , then go to step 1
14. Else, end of flooding.

#### Procedure $Z = \text{getFOZ}(m, w, h)$

1. Determine points such that  $w = \delta(h \wedge m)$  and store them in  $Z$ .
2. Calculate  $z$  (numerical convert of  $Z$ ).
3.  $v_0 = \text{vol}(Z)$ .
4.  $v_1 = v_0$
5. Compute  $\delta(w \wedge z)$
6. Put points such that  $\delta(w \wedge z) = w$  in  $Z$
7. Calculate  $z$

8.  $v_0 = \text{vol}(Z)$
9. If  $(v_0 > v_1)$  then go to step 4
10. End ( $Z$  contains the FOZ)

### **Mamba implementation**

[to be done]

### **Comments**

The above procedure requires some comments.

Firstly, it is more an ordering of the various catchment basins rather than a simulation of the flood. This procedure provides a kind of flooding graph which simply indicates which catchment basins (or unions of catchment basins) will be flooded next when a given catchment basin has been flooded. This corresponds to a hierarchy of catchment basins. However, the flooding/overflow procedure is synchronised. This means that the basins belonging to a same level of hierarchy will be flooded simultaneously only when all the catchment basins belonging to the previous level have been flooded (up to their respective FOZ). It is unlikely that a real flooding would follow this rule. We have, for instance, no control of the rate of flow in this procedure. This rate of flow will surely depend, in the real flooding process, of lots of factors: size of FOZ, dynamics of the flow (speed), etc. Moreover, when a small catchment basin is filled, neighbouring catchment basins will be flooded without waiting for the end of flooding of the other catchment basins belonging to the same hierarchy. Therefore, the calculation of the volume of water involved at each step of the above procedure may be biased and may not correspond to the real situation.

Secondly, this procedure does not simulate the flooding itself but it simply gives an idea of the result of this flooding, that is, the regions which could be affected by this flooding. Indeed, this information can be useful to establish risk maps.

Thirdly, the described procedure allows to know if the general flooding process is mainly achieved through a classical “watershed-like” flooding or through an overflow, by a very simple way illustrated at Fig 4-c (step 4): when at least two previously flooded regions merge, this means that the process is again a flooding process (a minimal catchment basin has been reached). On the contrary, if a new non connected flooded region appears, this region has been added through an overflow. At the end, the situation where the calculated volume of water corresponds to the real one involved is when the flooded surface is made of a single connected component as illustrated at Fig 1 (if only one flooding source is used).

It could be possible, then, to use the catchment basins hierarchy graph obtained by the above procedure to design another more realistic flooding schemes based on graph valuation and propagation. Although there exists morphological operators able to handle this kind of data structure (generalised geodesic operators), it is likely that these flooding schemes would also require the use of other mathematical and simulation tools.

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