SPATIAL STRUCTURES DETECTION IN HYPERSPECTRAL IMAGES USING MATHEMATICAL MORPHOLOGY

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ABSTRACT

The aim of this paper is to apply genuine hyperspectral mathematical morphology to extract spatial structures according to their spectral nature. To achieve this objective, a novel approach for vectorial ordering is introduced in this paper. The proposed ordering is based on a supervised framework which requires a reference spectrum for the image background and, at least, another reference spectrum for the image target. This supervised ordering may then used for the extension of mathematical morphology to vectorial images and in particular, we focus here on the application of morphological processing to hyperspectral images, illustrating the performance with real examples.

Index Terms— Mathematical Morphology, Supervised Learning, Spatial/Spectral Feature Extraction, Hyperspectral Imagery.

1. INTRODUCTION

Hyperspectral images (HSI) are acquired by a sensor that captures radiant data in typically hundreds of contiguous bands, producing a signature for each pixel in the image. However while, on the one hand, this increased spectral resolution makes possible an accurate detection and identification; on the other hand, the high dimensionality of data significantly increases the complexity of the analysis [1]. Usually the HSI analysis starts with a dimensional reduction stage followed by a definition of feature mappings to utilize machine learning techniques. In this kind of approach only the spectral information is considered. However, recent works have proven that accounting also for the local spatial information permits to increase the discrimination capability [2]. Our motivation in this paper is to analyze HSI using mathematical morphology (MM) operators, which deals naturally with the spatial structures, i.e., filtering out the image objects according to the geometry/shape and regional homogeneity.

Construction of morphological operators needs a complete lattice structure [3], i.e., the possibility of defining an ordering relationship between the points to be processed. From a theoretical viewpoint, a partial ordering is sufficient to construct complete lattices; however in practical algorithms we should require a total ordering (i.e., any pair of pixels must be totally ordered). Extending ordering to multivariate data is not straightforward, because there is no natural ordering in a vector space, as opposed to one-dimensional (scalar) case [4]. Therefore, the extension of MM to HSI, is neither direct nor trivial due to the vectorial nature of the data. For a general account on mathematical morphology the interested reader should refer to [5]. In HSI, some works [6, 7] have introduced nonlinear vectorial filtering preprocessing which the authors considered as the extension of morphological operators to vectorial images. These operators based on min/max cumulated distances, useful for various tasks are in fact vector median [8] and anti-median filters and cannot be referred as dilations and erosions in the mathematical morphology framework [9], i.e., the pair of vector median and vector anti-median are not adjunct operators and consequently they cannot be used to defined morphological filters as the openings/closings. Another clever alternative is proposed in [10] using morphological transformations to build morphological profiles (MP) applied marginally to principal component.

This paper introduces a supervised reduced orderings for vectorial spaces based in two reference pixels: a background spectrum and a target spectrum. We start by considering the theoretical framework of h-orderings, as introduced in [11], and then, the various families the h-mappings associated to reduced orderings are studied, to finally arrive to the notion of h-supervised ordering. The performance of the proposed morphological operators is illustrated with real examples from HSIs.

2. COMPLETE LATTICE IN $\mathbb{R}^P$ USING h-SUPERVISED ORDERING

A space $\mathcal{L}$ endowed with a partial order $\leq$ is called a *complete lattice* [3], denoted $(\mathcal{L}, \leq)$ if for every subset $\mathcal{H} \subseteq \mathcal{L}$ have both supremum (join) $\bigvee \mathcal{H}$ and infimum (meet) $\bigwedge \mathcal{H}$. A *minimum (smallest)* $n \in \mathcal{H}$ is an element contained in all other elements of $\mathcal{H}$, that is, $l \in \mathcal{H} \Rightarrow n \leq l$. We denote the minimum of $\mathcal{L}$ by $\perp$. Equivalently, a *maximum (largest)* n in $\mathcal{H}$ is an element that contains every element of $\mathcal{H}$, that is, $l \in \mathcal{H} \Rightarrow l \leq n$. We denote the maximum of $\mathcal{L}$ by $\top$. Let R be a nonempty set and $\mathcal{L}$ a complete lattice. Furthermore, let $h : R \rightarrow \mathcal{L}$ be a surjective mapping. As it was defined in [11], we refer by $\leq_h$ as the *h-ordering* given by:

$$r \leq_h r' \Leftrightarrow h(r) \leq h(r'), \quad \forall r, r' \in R$$

Note that $\leq_h$ preserves reflexivity ($r \leq_h r$) and transitivity ($r_1 \leq_h r_2$ and $r_2 \leq_h r_3 \Rightarrow r_1 \leq_h r_3$) but is not a total ordering.

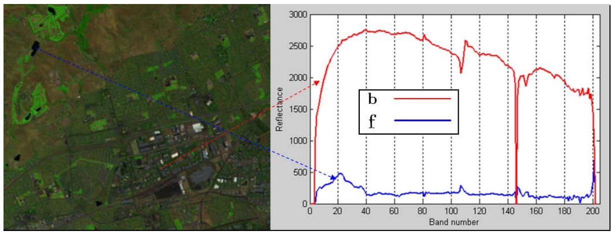
Reduced orderings. For multiband imagery, as color or hyperspectral images, pixel values are vectors defined in $\mathbb{R}^p$, consequently the main challenge to build complete lattice structures is to define a mapping $h : \mathbb{R}^p \to \mathcal{L}$, where $\mathcal{L}$ can be the lattice $(\mathbb{R}, \leq_{\mathbf{0}})$ using $\leq_{\mathbf{0}}$ as the "less than or equal to" operation. Once an ordering is defined for a set, the application of mathematical morphology operators is direct, and these operators are useful for denoising, object extraction and other tasks. Some authors have already worked in this idea. As it was noted in [4], two main families of mappings h for a given $\mathbf{x} = (x_1, x_2, \ldots, x_p) \in \mathbb{R}^p$ can be defined: 1) *based on projections* (unsupervised), i.e.; $h(\mathbf{x}) = \sum_{i=1}^{p} \lambda^i x_i$ That can be obtained by using the more representative projection in a statistical dimensional reduction technique, for example linear approaches as PCA, nonlinear approaches as Kernel-PCA or ISOMAP. 2) *based on distances* (supervised); given a subset $T \subset R$, $T = \{\mathbf{t}_1, \ldots, \mathbf{t}_{|T|}\}$, with $\mathbf{t}_i \in \mathbb{R}^p, \forall i$; $h(\mathbf{x})$ can be written as $h(\mathbf{x}) = \sum_{i=1}^{|T|} \lambda^i \phi(\mathbf{t}_i, \mathbf{x})$ where $\phi : \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}^+$ is a vectorial distance. This idea has been exploited for instance in [9] using a colour of reference. The Mahalanobis distance has been employed in several works on multivariate morphology [11] [12]. We must remark that in these methodologies the reference spectrum, or the distribution of reference, are associated to the "image foreground" (or image target). That means that dilation tends to approach the pixels towards the reference, and by duality, the erosion tends to move further away the reference, but without defining to which "image background" the spectrum must be addressed. This *asymmetric* situation is one the problems that are considered in this paper. In sequel, in the standard case a single vector of reference $\mathbf{f}$, we define h as $h_{\mathbf{f}}(\mathbf{x}) = -K(\mathbf{f}, \mathbf{x})$.

***h*-supervised ordering.** Let us focus on the case of h-ordering based on distances. We defined a *h-supervised ordering* for a nonempty set R based on the elements $b, f \in R$, such that $b \neq f$ is a h-ordering that satisfies the conditions: $h(\mathbf{f}) = \top$ and $h(\mathbf{b}) = \bot$. Note that $\bot, \top$ are the smallest and largest element in the lattice $\mathcal{L}$.

By the limited length of the paper we skip the details, but we consider in our approach that the h-supervised ordering is induced by

$$h_{\{\mathbf{b},\mathbf{f}\}}(\mathbf{x}) = \frac{K(\mathbf{f}, \mathbf{x}) - K(\mathbf{b}, \mathbf{x})}{K(\mathbf{x}, \mathbf{x}) - K(\mathbf{f}, \mathbf{b})} \qquad (1)$$

where K is a positive definite kernel. Full calculations using Kriging interpolation and SVM which leads to expression 1 are given in [13]. This expression handles the relation between background and foreground pixels ($\mathbf{b}$,$\mathbf{f}$) for all vectors $\mathbf{x}$. Supervised reduced ordering is basically the difference between kernelized distances. Note that if $K(\mathbf{x}, \mathbf{y})$ has an unique maximum in $\mathbf{x} = \mathbf{y}$ then $K(\mathbf{x}, \mathbf{x}) > K(\mathbf{f}, \mathbf{b})$ and therefore $\mathbf{x} \to \mathbf{b} \Rightarrow h(\mathbf{x}) \to 1$ and if $\mathbf{x} \to \mathbf{f} \Rightarrow h(\mathbf{x}) \to -1$, where "→" means "tend to".



(a) Original Image $\mathbf{I}$ and pixels $\mathbf{b}$, $\mathbf{f}$

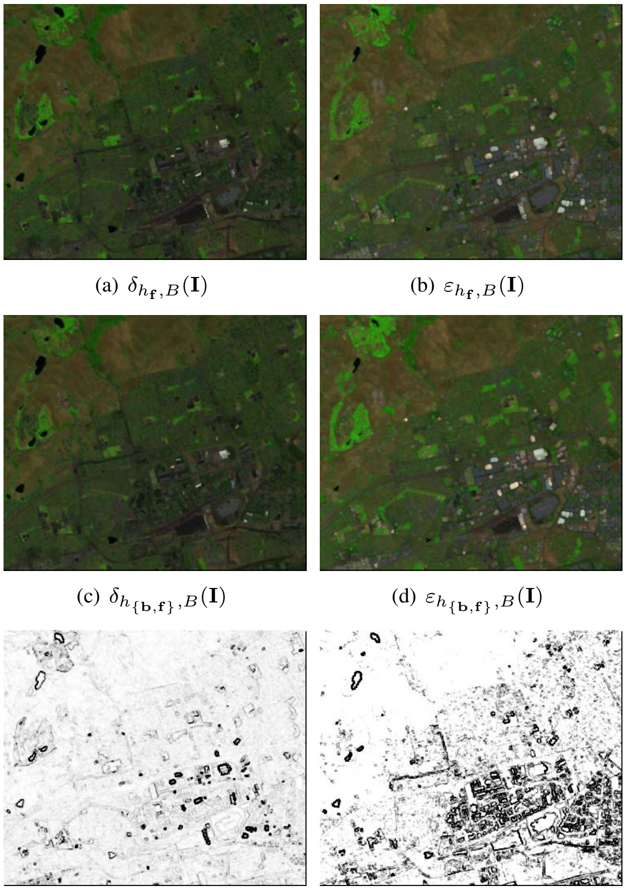
Fig. 1. HSI Moffett Field subscene using bands {115,70,30}. Reference pixels used in the experiments.

3. MORPHOLOGICAL OPERATORS IN VECTORIAL SPACES

Once the family of orderings have been established, the morphological vector operators are defined in the standard way. We limit here our developments to the flat operators, i.e., the structuring elements are planar. We denote by $\mathcal{F}(E, \mathcal{L})$ the functions from E onto $\mathcal{L}$. If $\mathcal{L}$ is a complete lattice, then $\mathcal{F}(E, \mathcal{L})$ is a complete lattice too. We need to recall a few notions which characterize the properties of morphological operators. Let ψ be an operator on a complete lattice $\mathcal{F}(E, \mathcal{L})$. ψ is increasing if $\forall \mathbf{I}, \mathbf{G} \in \mathcal{F}(E, \mathcal{L})$, $\mathbf{I} \leq_h \mathbf{G} \Rightarrow \psi(\mathbf{I}) \leq_h \psi(\mathbf{I})$. It is anti-extensive if $\psi_h(\mathbf{I}) \leq_h \mathbf{I}$ and it is if $\mathbf{I} \leq_h \psi_h(\mathbf{I})$. An operator is idempotent if it is verified that $\psi(\psi(\mathbf{I})) = \psi(\mathbf{I})$. The *supervised erosion* of an image $\mathbf{I} \in \mathcal{F}(E, \mathcal{L})$ at pixel $x \in E$ by the structuring element $B \subset E$ of size n is given by $\varepsilon_{h,nB}(\mathbf{I})(x) = \{\mathbf{I}(y) : \mathbf{I}(y) = \wedge_h[\mathbf{I}(z)], z \in n(B_x)\}$, where $\wedge_h$ is the infimum according to the total ordering h. The corresponding *supervised dilation* δ_h, is obtained by replacing the $\inf_h$ by the $\sup_h$, i.e., $\delta_{h,nB}(\mathbf{I})(x) = \{\mathbf{I}(y) : \mathbf{I}(y) = \vee_h[\mathbf{I}(z)], z \in n(B_x)\}$. The erosion and the dilation are increasing operators. Moreover, the erosion is anti-extensive and the dilation is extensive. In practice, the supervised erosion shrinks the structures which have a spectrum close to the foreground; "peaks of spectrum" thinner than the structuring element disappear by taking the spectrum of neighboring structures with a spectrum values close to the background. As well, it expands the structures which have a vector value close to background. Dilation produces the dual effects, enlarging the regions having a spectrum close to the foreground and contracting the background. A *supervised opening* is an erosion followed by a dilation, i.e., $\gamma_{h,nB}(\mathbf{I}) = \delta_{h,nB}(\varepsilon_{h,nB}(\mathbf{I}))$, and a *supervised closing* is a dilation followed by an erosion, i.e., $\varphi_{h,nB}(\mathbf{I}) = \varepsilon_{h,nB}(\delta_{h,nB}(\mathbf{I}))$. The opening (closing) is an anti-extensive (extensive) operator. More precisely, the opening removes spectrum peaks that are thinner than the structuring element, having a vector value close to the foreground; the closing remove vector values peaks that are thinner than the structuring element, having a spectrum close to background. Moreover, using a the h-supervised ordering to calculate the image distance $d \in \mathcal{F}(E, \mathcal{L})$ (a scalar function), given by the difference point-by-point of two hyperspectral images $d_h(\mathbf{I}, \mathbf{G})(x) = h(\mathbf{I}(x)) - h(\mathbf{G}(x))$, we can easily define the *supervised morphological gradient*, i.e.,$\varrho_h(\mathbf{I}) = d_h(\delta_{h,B}(\mathbf{I}), \varepsilon_{h,B}(\mathbf{I}))$. This function gives the contours of the image, attributing more importance to the transitions between regions close/far to the background/foreground. The *positive supervised top-hat transformation* is the residue of a supervised opening, i.e.,$\rho^{+}_{h,nB}(\mathbf{I}) = d_h(\mathbf{I}, \gamma_{h,nB}(\mathbf{I}))$. Dually, *negative supervised top-hat transformation* is given by $\rho^{-}_{h,nB}(\mathbf{I}) = d_h(\varphi_{h,nB}(\mathbf{I}), \mathbf{I})$. The top-hat transformation yields grey level images and is used to extract contrasted components with respect to the background. Moreover, top-hats remove the slow trends, and thus enhancing the contrast of objects smaller than the structuring element used for the opening/closing. For lack of space, we have to give up the details about geodesic reconstruction, levelings, alternate sequential filters and so on, but they are natural extensions using the h-ordering and d_h distance. However, additional examples of our approach using also geodesic operators can be found in the website [1].

4. APPLICATIONS TO HSI

In practice, a total order is required to avoid random decisions in lattice operations. In sequel, the partial h-ordering is completed with a lexicographic ordering ($<^{d}$) to have a total ordering, in $\mathcal{L}[z]$, for all z, and it is denoted as $<_h$. To illustrate the application of MM in HSIs, an experiment was performed using imagery collected by the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) developed at the Jet Propulsion Laboratory of NASA. This sensor operates in the Visible to Near Infrared and Short-Wave Infrared range of 400 nm to 2.5 m sampled to achieve 224 spectral bands. To test the ability to detect targets with a-priori information of size, experiments using an AVIRIS HSIs from the Moffett Field data set is performed. The number of spectral bands is initially reduced to 203 by removing extremely noisy bands, i.e., we include the information contained in bands [1:106,114:153,168:224]. Figure 1(a) is the false-color preview for the 370 columns, 313 rows in 203 bands. A set of two pixels are selected for foreground(water) and background(building) as it is showed in Figure 1(a). An hexagonal unitary structuring element B and a polynomial kernel K of degree 2 are used. Supervised erosion and dilation (Figure 2) do not include false spectrum and they can be interpreted as usual morphological operators, i.e., pixels are selected in the structuring element neighborhood using the ordering induced by $h_{\mathbf{f}}$ and $h_{\{\mathbf{f},\mathbf{b}\}}$, correspondingly. Supervised morphological gradient in $h_{\mathbf{f}}$ (Fig. 2(e)) is favoring structures with high differences among contiguous pixels. Supervised morphological gradient based on $h_{\{\mathbf{b},\mathbf{f}\}}$ highlights spatial pattern favoring structures containing the reference pixels $\mathbf{b}$ or $\mathbf{f}$ (Fig. 2(f)). Positive supervised top-hat transformations are presented in figure 3 which allows to extract regions with spectrum close to foreground (water) and with size is smaller than the correspondent structuring element. Dually, negative supervised top-hat transformations (Figure 3(b)) emphasize information according background/size. In the case of $h_{\mathbf{f}}$, (Fig. 3(a)), interpretation is not clear due to a lack of duality in ordering induced for a single pixel $\mathbf{f}$. That example illustrates the main advantage to consider a second pixel of reference to obtain a supervised ordering. Additional examples can be found in [1].


(a) $\delta_{h_{\mathbf{f}},B}(\mathbf{I})$ (b) $\varepsilon_{h_{\mathbf{f}},B}(\mathbf{I})$

(c) $\delta_{h_{\{\mathbf{b},\mathbf{f}\}},B}(\mathbf{I})$ (d) $\varepsilon_{h_{\{\mathbf{b},\mathbf{f}\}},B}(\mathbf{I})$

(e) $\varrho_{h_{\mathbf{f}}}$ (f) $\varrho_{h_{\{\mathbf{b},\mathbf{f}\}}}$

Fig. 2. Supervised dilation (a)(c) and dilation (b)(d) using two supervised ordering. Supervised morphological gradient images for ordering induced by $h_{\mathbf{f}}$ (e) and $h_{\{\mathbf{b},\mathbf{f}\}}$(f)

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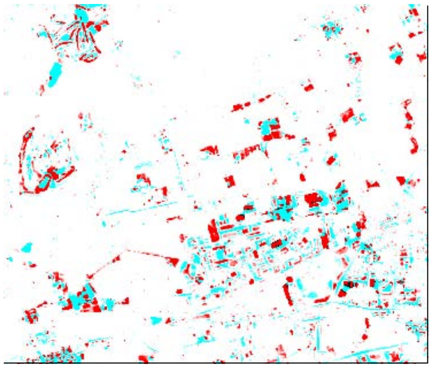
[1] `http://cmm.ensmp.fr/∼velasco/SuperHSI`



(a) $\rho^{+}_{h_{\mathbf{f}},5B}(\mathbf{I})$, $\rho^{-}_{h_{\mathbf{f}},5B}(\mathbf{I})$

(b) $\rho^{+}_{h_{\{\mathbf{f},\mathbf{b}\}},5B}(\mathbf{I})$, $\rho^{-}_{h_{\{\mathbf{f},\mathbf{b}\}},5B}(\mathbf{I})$

Fig. 3. Positive and negative supervised top-hat transformation for ordering induced by $h_{\mathbf{f}}$(a) and $h_{\{\mathbf{b},\mathbf{f}\}}$(b)

5. CONCLUSIONS AND PERSPECTIVES

Hyperspectral imaging is an active field of image analysis which is usually considered under the supervised paradigm: both the complexity of the data and the typical real-life applications require the construction of training sets of spectra which drive the algorithms of classification, feature extraction, segmentation, etc. Hence, from our viewpoint, the construction of hyperspectral mathematical morphology operators should be also coherent with this idea of supervised processing. Our formulation based on both a background pixel and a foreground pixel allows an adequate interpretation of dual morphological operations. We have also shown that the practical requirement of a total ordering, for instance in hyperspectral image processing, makes necessary the use of some arbitrary information (lexicographic cascades), but playing only in a very secondary role in the data ordering. The first examples given in the paper illustrate the potential interest of the algorithms; however more advanced applications are currently under study. Despite of the limited information introduced, only two pixels, morphological operation are able to extract spatial pattern in the considered hyperspectral image without apply any dimensional reduction algorithm.

6. REFERENCES

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