UNIFIED MORPHOLOGICAL COLOR PROCESSING FRAMEWORK IN A LUM/SAT/HUE REPRESENTATION

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Abstract The extension of lattice based operators to color images is still a challenging task in mathematical morphology. The first choice of a well-defined color space is crucial and we propose to work on a lum/sat/hue representation in norm L_1. We then introduce an unified framework to consider different ways of defining morphological color operators either using the classical formulation with total orderings by means of lexicographic cascades or developing new transformations which takes advantage of an adaptive combination of the chromatic and the achromatic (or the spectral and the spatio-geometric) components. More precisely, we prove that the presented saturation-controlled operators cope satisfactorily with the complexity of color images. Experimental results illustrate the performance and the potential applications of the new algorithms.

Keywords: color mathematical morphology, luminance/saturation/hue, lexicographic orderings, reconstruction, gradient, top-hat, leveling, segmentation

1. Introduction

Mathematical morphology is the application of lattice theory to spatial structures [16] (i.e. the definition of morphological operators needs a totally ordered complete lattice structure). Therefore the extension of mathematical morphology to color images is difficult due to the vectorial nature of the color data. Fundamental references to works which have formalized the vector morphology theory are [17] and [8].

Here we propose here a unified framework to consider different ways of defining morphological color operators in a luminance, saturation and hue color representation. This paper is a summary of the Ph. D. Thesis of the author [1] done under the supervision of Prof. Jean Serra (full details of the algorithms and many other examples can be found in [1]).

2. Luminance/Saturation/Hue color in norm L_1

The primary question to deal with color images involves choosing a suitable color space representation for morphological processing. The RGB color representation has some drawbacks: components are strongly correlated, lack of human interpretation, non uniformity, etc. A polar representation with the variables luminance, saturation et hue (lum/sat/hue) allows us to solve these problems. The HLS system is the most popular lum/sat/hue triplet. In spite of its popularity, the HLS representation often yields unsatisfactory results, for quantitative processing at least, because its luminance and saturation expressions are not norms, so average values, or distances, are falsified. In addition, these two components are not independent, which is not appropriate for a vector decomposition. The reader can find a comprehensive analysis of this question by Serra [20]. The drawbacks of the HLS system can be overcome by various alternative representations, according to different norms used to define the luminance and the saturation. The L_1 norm system has already been introduced in [18] as follows:

$$\left\{ \begin{array}{l} l = \frac{1}{3}(max + med + min) \\ s = \left\{ \begin{array}{ll} \frac{3}{2}(max - l) & if\ \ l \geq med \\ \frac{3}{2}(l - min) & if\ \ l \leq med \end{array} \right. \\ h = k\left[\lambda + \frac{1}{2} - (-1)^{\lambda}\left(\frac{max+min-2med}{2s}\right)\right] \end{array} \right. \tag{1}$$

where max, med and min refer the maximum, the median and the minimum of the RGB color point (r, g, b), k is the angle unit ($\pi/3$ for radians and 42 to work on 256 grey levels) and $\lambda = 0,$ if $r > g \geq b$; $1,$ if $g \geq r > b$; $2,$ if $g > b \geq r$; $3,$ if $b \geq g > r$; $4,$ if $b > r \geq g$; $5,$ if $r \geq b > b$ allows to change to the color sector. In all processing that follows, the l, s and h components are always those of the system (1), named LSH representation.

3. Morphological color operators from LSH

For detailed exposition on complete lattice theory refer to [7]. Let $E, \mathcal{T}$ be nonempty set. We denote by $\mathcal{F}(E, \mathcal{T})$ the power set $\mathcal{T}^E$, i.e., the functions from E onto $\mathcal{T}$. If $\mathcal{T}$ is a complete lattice, then $\mathcal{F}(E, \mathcal{T})$ is a complete lattice too. Let f be a grey level image, $f : E \rightarrow \mathcal{T}$, in this case $\mathcal{T} = \{t_{min}, t_{min} + 1, \cdots, t_{max}\}$ is an ordered set of grey-levels. Given the three sets $\mathcal{T}^l, \mathcal{T}^s, \mathcal{T}^h$, we denote by $\mathcal{F}(E, [\mathcal{T}^l \otimes \mathcal{T}^s \otimes \mathcal{T}^h])$ or $\mathcal{F}(E, \mathcal{T}^{lsh})$ all color images in a LSH representation ($\mathcal{T}^{lsh}$ is the product of $\mathcal{T}^l, \mathcal{T}^s, \mathcal{T}^h$, i.e., $\mathbf{c}_i \in \mathcal{T}^{lsh} \Leftrightarrow \mathbf{c}_i = \{(l_i, s_i, h_i); \, l_i \in \mathcal{T}^l, s_i \in \mathcal{T}^s, h_i \in \mathcal{T}^h\}$). We denote the elements of $\mathcal{F}(E, \mathcal{T}^{lsh})$ by $\mathbf{f}$, where $\mathbf{f} = (f_L, f_S, f_H)$ are the color component functions. Using this representation, the value of $\mathbf{f}$ at a point

$x \in E$, which lies in $\mathcal{T}^{lsh}$, is denoted by $\mathbf{f}(x) = (f_L(x), f_S(x), f_H(x))$. Note that the sets $\mathcal{T}^l, \mathcal{T}^s$ corresponding to the luminance and the saturation are complete totally ordered lattices. The hue component is an angular function defined on the unit circle, $\mathcal{T}^h = \mathcal{C}$, which has no partial ordering. Let $a : E \to \mathcal{C}$ be an angular function, the angular difference [15, 9] is defined as $a_i \div a_j = \mid a_i - a_j \mid$ if $\mid a_i - a_j \mid \leq 180^o$ or $a_i \div a_j = 360^o - \mid a_i - a_j \mid$ if $\mid a_i - a_j \mid > 180^o$. It is possible to fix an origin on $\mathcal{C}$, denoted by h_0. We can now define a h_0-centered hue function by computing $f_H(x) \div h_0$. The function $(f_H \div h_0)(x)$ is an ordered set and therefore leads to a total complete ordered lattice denoted by $\mathcal{T}^{h \div h_0}$. We propose to distinguish two main classes of morphological color operators, the vector-to-vector operators or VV-operators and the vector-to-scalar operators or VS-operators. Let $\mathbf{f}, \mathbf{g} \in \mathcal{F}(E, \mathcal{T}^{lsh})$ be two color images in LSH color space and $h \in \mathcal{F}(E, \mathcal{T})$ a grey level image. An operator Ψ is called a VV-operator if $\Psi : \mathcal{T}^{lsh} \to \mathcal{T}^{lsh};\ \mathbf{g} = \Psi(\mathbf{f})$. An operator Φ is called a VS-operator if $\Phi : \mathcal{T}^{lsh} \to \mathcal{T};\ h = \Phi(\mathbf{f})$. In addition, a connective criterion $\sigma : \mathcal{F} \otimes \mathcal{P}(E) \to [0, 1]$ can be applied to a color image $\mathbf{f}$ for segmenting and obtaining a partition D_σ (see Serra's segmentation theory [19]). For the sake of simplicity, we consider that a segmentation operator based on a connective criterion is a VS-operator. Different ordering relationships between vectors have been studied [5]. The marginal ordering or M-ordering is a partial ordering, based on the usual pointwise ordering (i.e., component by component). Another more interesting ordering is called conditional ordering or C-ordering, where the vectors are ordered by means of some marginal components sequentially selected according to different conditions. This is commonly named as lexicographic ordering which is a total ordering. Using a M-ordering for the elements of $\mathcal{F}(E, \mathcal{T}^{lsh})$ we can introduce color vector values in the transformed image that are not present in the input image (the problem of "false colors"). The application of a C-ordering in $\mathcal{F}(E, \mathcal{T}^{lsh})$ preserves the input color vectors. When dealing with operators for color images $\mathcal{F}(E, \mathcal{T}^{lsh})$ C-orderings are indicated to build VV-operators ($\mathbf{g} = \Psi(\mathbf{f})$), introducing no new colors [21], but can be also used for VS-operators ($h = \Phi(\mathbf{f})$). An inconvenient of the C-orderings (vectorial approach) is the computational complexity of the algorithms which leads to slow implementations. However, in practice, for many applications (e.g. segmentation and feature extraction) involving VS-operators, total orderings are not required as well as increment based operators (e.g. gradients and top-hats) can be defined in the unit circle $\mathcal{T}$ without fixing an origin on the hue component [9]. We consider therefore that M-orderings can be interesting for developing color operators. Let ψ_i be the mapping $\psi_i : \mathcal{T} \to \mathcal{T}$ an operator for grey level images (marginal operator). In general, a separable marginal operator is formalized by $h = \Xi\left(\Psi_1(f_L), \Psi_2(f_S), \Psi_3(f_H)\right)$, where Ξ is a merging function (linear or non-linear) to combine the components. Obviously, although less useful, M-operators can be also applied to VV-operators (i.e., $\mathbf{g} = (\Psi_1(f_L), \Psi_2(f_S), \Psi_3(f_H))$).

4. Total orderings using lexicographical cascades

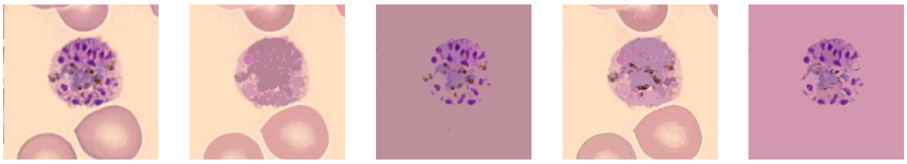
Let $\mathbf{c}_i = (u_i, v_i, w_i)$ and $\mathbf{c}_j = (u_j, v_j, w_j)$ be two arbitrary color points, i.e., $\mathbf{c}_i, \mathbf{c}_j \in \mathcal{T}^{lsh}$, where the generic components (u_k, v_k, w_k) are $f_L(x)$, $f_L(x)$ the and negative of the h_0-centered hue $(f_H(x) \div h_0)$ (the closest value $f_H(x)$ to h_0 must be the supremum) of the color image $\mathbf{f}$ at point x. The Ω-lexicographical ordering or $<_\Omega$ is defined as

$$\mathbf{c}_i <_\Omega \mathbf{c}_j \ if \ \begin{cases} u_i < u_j & or \\ u_i = u_j & and \quad v_i < v_j \quad or \\ u_i = u_j & and \quad v_i = v_j \quad and \quad w_i < w_j \end{cases}$$

We denote the lexicographical cascade by Ω_{uvw}. In this case the priority is given to the component u, then to v and finally to w. Obviously, it is possible to define other orderings for imposing a dominant role to any other of the vector components. The drawback of this kind of orderings is that most of vector pairs are sorted by the chosen first component. There is a simple way in order to make Ω-ordering more flexible which involves the linear reduction of the dynamic margin of the first component, applying a division by a constant and rounding off, i.e., changing u by $\lceil u/\alpha \rceil$. It is named an $\alpha-$modulus Ω-lexicographical ordering. The value for α controls the influence degree of the first component with regard to the others (above all the second one, since the cascade almost never reaches the third row).

We then define three main families of lexicographical orderings from the representation LSH: luminance-based $\Omega_{l|_\alpha s(h \div h_0)}$, saturation-based $\Omega_{s|_\alpha l(h \div h_0)}$ and hue-based $\Omega_{(h \div h_0)sl}$. The value of h_0 yields an important degree of freedom which allows us to act on a specific hue. A disadvantage of the hue-based ordering is its instability for the low saturation points (different solutions can be used which are based on a weighting of the hue by the saturation [1]). The C-ordering color morphology has been widely studied in the framework of lum/sat/hue representations such as we propose here (e.g. by Hanbury and Serra [10], by Ortiz et al. [14]), but most of works are very preliminary studies, being limited to the basic operators

Once these orders have been defined, the morphological color VV-operators are defined in the standard way. We limit our developments to the flat operators. The color erosion of an image $\mathbf{f}$ at pixel x by the structuring element B of size n is $\varepsilon_{\Omega, nB}(\mathbf{f})(x) = \{\mathbf{f}(y) : \mathbf{f}(y) = \inf_\Omega [\mathbf{f}(z)], z \in$



$\mathbf{f}$ $\varphi^{rec}_{\Omega_1,n_1B}(\mathbf{f})$ $\gamma^{rec}_{\Omega_1,n_2B}(\mathbf{f})$ $\varphi^{rec}_{\Omega_2,n_1B}(\mathbf{f})$ $\varphi^{rec}_{\Omega_3,n_2B}(\mathbf{f})$

Figure 1. Detection of inclusions in erythrocytes (malaria diagnosis) using color openings/closings by reconstruction. The LSH lexicographical orderings are $\Omega_1 = \Omega_{ls(h \div h_0)}$, $\Omega_2 = \Omega_{(h \div h_0)sl}$ with $h_0 = 90$ (green-yellow, opposite to blue-purple), $\Omega_3 = \Omega_{(h \div h_0)sl}$ with $h_0 = 270$ (blue-purple) ; and where $n_1 = 15$, $n_2 = 200$, B is an unit square SE.

$n(B_x)\}$, where $\inf_\Omega$ is the infimum according to the lexicographical ordering Ω. The corresponding color dilation $\delta_{\Omega,nB}$ is obtained by replacing the $\inf_\Omega$ by the $\sup_\Omega$. A color opening $\gamma_{\Omega,nB}$ is an erosion followed by a dilation, and a color closing $\varphi_{\Omega,nB}$ is a dilation followed by an erosion. Once the color opening and closing are defined it is obvious how to extend other classical operators like the alternate sequential filters or the granulometries. Moreover, using a vectorial distance to calculate the difference point-by-point of two images $d(\mathbf{f},\mathbf{g})(x)$, $d : \mathcal{T}^{lsh} \times \mathcal{T}^{lsh} \rightarrow \mathcal{T}$, $x \in E$, we can easily define the two most classical VS-operators: the morphological gradient, i.e., $\varrho_\Omega(\mathbf{f}) = d(\delta_{\Omega,B}(\mathbf{f}), \varepsilon_{\Omega,B}(\mathbf{f}))$, and the top-hat transformation, i.e., $\rho^+_{\Omega,nB}(\mathbf{f}) = d(\mathbf{f}, \gamma_{\Omega,nB}(\mathbf{f}))$. In addition, we propose also the extension of the operators "by reconstruction" implemented using the color geodesic dilation which is based on restricting the iterative dilation of a function marker $\mathbf{m}$ by B to a function reference $\mathbf{f}$ [22], i.e., $\delta^n_\Omega(\mathbf{m},\mathbf{f}) = \delta^1_\Omega\delta^{n-1}_\Omega(\mathbf{m},\mathbf{f})$, where $\delta^1_\Omega(\mathbf{m},\mathbf{f}) = \delta_{\Omega,B}(\mathbf{m}) \wedge_\Omega \mathbf{f}$. The color reconstruction by dilation is defined by $\gamma^{rec}_\Omega(\mathbf{m},\mathbf{f}) = \delta^i_\Omega(\mathbf{m},\mathbf{f})$, such that $\delta^i_\Omega(\mathbf{m},\mathbf{f}) = \delta^{i+1}_\Omega(\mathbf{m},\mathbf{f})$ (idempotence). In a similar way the color leveling $\lambda_\Omega(\mathbf{m},\mathbf{f})$ is computed by means of an iterative algorithm with geodesic dilations and geodesic erosions until idempotence [12].

In figure 1 an example of application for the detection of inclusions in red blood cells (parasites of the malaria Plasmodium Vivax) is given. The inclusions are two types of dark structures: blue-purple ones and brown others, which can be detected separately. Using openings/closings by reconstruction on $\Omega_{ls(h \div h_0)}$ ordering, all inclusions are removed/enhanced together whereas choosing the adequate h_0 angle, the hue-based ordering allows a more specific selection of the blue-purples ones. Note also that working on the hue-based $\Omega_{(h \div h_0)sl}$ ordering, it is possible to use a color closing to remove or to enhance the structures according to the hue origin (h_0 is the color of the structure or h_0 is the opposite on $\mathcal{C}$). An-

$\mathbf{f}$ $\rho^C_B(\mathbf{f})$ $\rho^{\downarrow}_B(\mathbf{f})$ $\rho^{A-}_B(\mathbf{f})$

Figure 2. Color top-hat's for detail extraction in cartographic image.

other example of color filtering is shown in figure 4: a color leveling using a luminance-based α-modulus ordering to simplify the texture/contours of the image [2] (the value of $\alpha = 10$ has shown to achieve robust and pleasant levelings) where the marker is a median filter of size 11×11.

5. Marginal orderings and merging by saturation-controlled operators

The saturation s is associated to the intensity of the hue h and has the intrinsic role of discrimination of the color points as chromatic (high s value) or achromatic (low s value). In this section, we discuss how to define marginal separable saturation-controlled VS-operators which cope satisfactorily with the complexity of color images. We propose also a hybrid VS- and VV-operator to filter adaptively color images. We suppose here f_S is normalized between 0 and 1.

Color top-hats for feature extraction

In the sense of Meyer [11], there are two versions of the top-hat for numerical functions ($f : E \to \mathcal{T}$). The white top-hat is the residue of the initial image f and an opening $\gamma_B(f)$, i.e. $\rho^+_B(f) = f - \gamma_B(f)$ (extracting bright structures) and the black top-hat is the residue of a closing $\varphi_B(f)$ and f, i.e. $\rho^-_B(f) = \varphi_B(f) - f$ (extracting dark structures). This numerical residue involves increments and hence can be defined to circular functions as the hue component. The circular centered top-hat [9] of an angular function is defined by $\rho^\circ_B(a(x)) = -\sup\{\inf[a(y) \div a(x), y \in B(x)]\}$ (extracting fast angular variations). Starting from these grey-level transformations, let us propose a series of definitions for the top-hat of a color image $\mathbf{f}$ from a LSH representation. The chromatic top-hat is given by $\rho^C_B(\mathbf{f}) = [f_S \times \rho^\circ_B(f_H)] \vee \rho^+_B(f_S)$. This operator extracts the fast variations of color regions on a saturated color background (i.e. saturated color peaks on uniform color regions) and the fast variations of saturated color regions on an achromatic (unsaturated) background (i.e. saturated

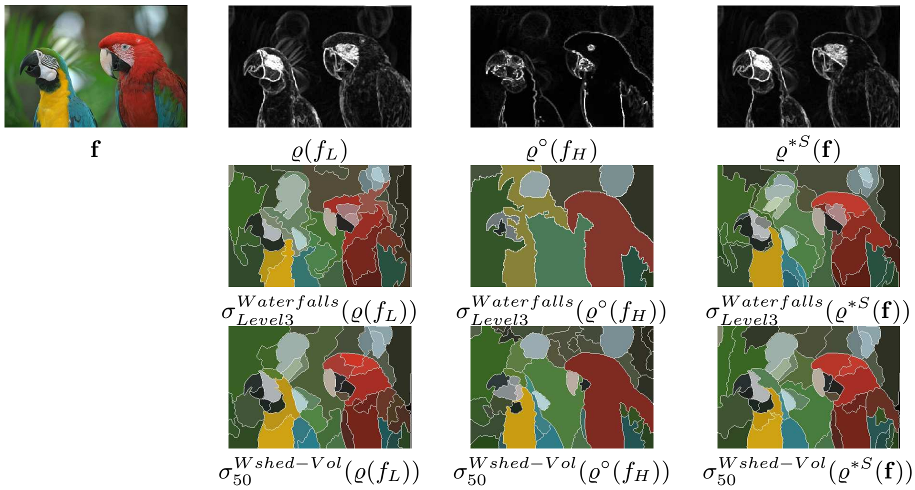


Figure 3. Color gradients and segmentation by watershed transformation.

color peaks on achromatic regions). The white-achromatic top-hat is the difference between the chromatic top-hat and the global bright top-hat, $\rho^{A+}_B = \rho^C_B - \rho^{\uparrow}_B$, where the global bright top-hat is calculated by $\rho^{\uparrow}_B(\mathbf{f}) = \rho^+_B(f_L) \vee \rho^-_B(f_S)$. It characterises the fast variations of bright regions (i.e. positive peaks of luminance) and the fast variations of achromatic regions on a saturated background (i.e. unsaturated peaks: black, white and grey on color regions). The black-achromatic top-hat is the difference $\rho^{A-}_B = \rho^C_B - \rho^{\downarrow}_B$, where the global dark top-hat is obtained by $\rho^{\downarrow}_B(\mathbf{f}) = \rho^-_B(f_L) \vee \rho^-_B(f_S)$. Dually, it copes with the fast variations of dark regions (i.e. negative peaks of luminance) and the fast variations of achromatic regions on a saturated background. The term $\rho^-_B(f_S)$ appears in both $\rho^{\uparrow}_B$ and $\rho^{\downarrow}_B$ to achieve symmetrical definitions. Figure 2 shows the color top-hats of a cartographic image [4]. The extracted objects are different and certain kinds of details are better defined on one top-hat than on the other. Their contributions are consequently complementary.

Color gradient for segmentation

The morphological gradient by Beucher [6] is the numerical residue of a dilation and an erosion, i.e., $\varrho(f) = \delta_B(f) - \varepsilon_B(f)$ (where B is an unitary disk). In a similar way as for the top-hat, a version has been defined for the angular functions. The circular centered gradient is given by [9] $\varrho^{\circ}(a(x)) = \vee[a(x) \div a(y), y \in B(x)] - \wedge[a(x) \div a(y), y \in B(x)]$.

We introduce the color gradient from a LSH representation by means of the following barycentric merging function: $\varrho^{*S}(\mathbf{f}) = f_S \times \varrho^{\circ}(f_H) + (1 - f_S) \times \varrho(f_L)$. The watershed transformation, a pathwise connection, is one of the most powerful tools for segmenting images. Typically, the function to flood is a gradient function which yields the transitions between the regions. The color gradient $\varrho^{*S}(\mathbf{f})$ may therefore be used for segmenting color images. Figure 3 depicts a comparative example of the partitions obtained by watershed algorithms using different gradients. The approach $\sigma^{Waterfalls}_{Level3}$ is the level 3 of a non-parametric pyramid of watershed (waterfalls algorithm [6]) and the $\sigma^{Wshed-Vol}_{50}$ is a marker-based watershed by selecting the 50 minima of highest volume extinction value [13]. Using either of the methods, the results obtained by means of the saturation weighting-based color gradient are better than working only on the luminance or on the hue gradient and even better than taking as color gradient the supremum of the three marginal gradients [3]. Based on a similar paradigm, the saturation, considered as a binary key, can be also used for merging the partitions associated to the hue and the luminance (see also [3]).

Regional-based color leveling for simplification

Finally, we shall introduce a regional-based color leveling algorithm. The rationale behind this technique is to work on two steps. First, to obtain a partition of the image $D_\sigma(\mathbf{f}) = \{R_i\}_{i=1}^{n}$ using the precedent color segmentation algorithm. Now, according the mean value of the saturation in each region R_i, the region is classified as chromatic or achromatic, and then, in the second step, each color image region $R_i(\mathbf{f})$ is independently leveled with $\lambda_{\Omega_{(h\div h_0)sl}}$ or $\lambda_{\Omega_{ls(h\div h_0)}}$ respectively (the marker is the median color of $R_i(\mathbf{f})$). In fact, this technique is an example of combination of a M-ordering operator (color gradient) followed by a C-ordering (leveling) adapted to the nature of the region. The results obtained by this filtering approach (see example in figure 4) yields very strong simplifications (in terms of color flat zones reduction) but keeping enough visual information. Consequently, it can be useful for region-based coding applications.

$\mathbf{f}$ $\quad$ $\lambda_\Omega(VMF_n(\mathbf{f}), \mathbf{f})$ $\quad$ $\lambda_{D_\sigma}(\mathbf{f})$

Figure 4 Morphological simplification of a color image. Comparison of a color levelling and a regional-based color levelling.

6. Conclusions and perspectives

The extension of mathematical morphology operators to multi-valued functions is neither direct nor general, above all if the aim is to obtain useful transformations. We have focused on color images in order to develop specific well-adapted morphological color operators. To achieve that, we have proceeded in three steps. ▷1 Use of a color representation (system LSH in norm L_1) which yields: (i) a correct formalization from a mathematical viewpoint, (ii) an intuitive interpretation of effects (as it is usual in mathematical morphology); ▷2 Explore the direct extension of morphological operators by using lexicographic orderings on the LSH system, proving the improvement for filtering applications when compared to the use of luminance only; ▷3 Introduce new marginal operators which take advantage of an adaptive combination of the chromatic and the achromatic (or the spectral and the spatio-geometric) components. Moreover, these separable mechanisms allow the application of classical grey level implementations with simple complexity elements to be added.

We can conclude that the dichotomy C-ordering vs. M-ordering for color operators can be integrated in an unified framework providing a wide range of operators. We have demonstrated by means of different applications on real images (biomedical microscopy, cartography, segmentation and coding in multimedia, etc.) the advantages of our new algorithms. We believe that the proposed methodology opens new possibilities for the application of mathematical morphology to color. Especially, we are working on three issues: (i) geodesic color reconstruction for specific object extraction, (ii) skeletons and thinnings of color objects, (iii) color granulometries.

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