Course "Physics and Mechanics of Random Media"
Ecole des Mines de Paris
(19-23 March 2012)

Room to be defined

Keywords: random structures, variability, simulations, homogenization, upscaling, elasticity, fracture statistics, reliability, computer aided design of materials

Lecturers:

Michel Bornert, Laboratoire Navier, Équipe « Multi-échelle », École des Ponts ParisTech, Champs-sur-Marne.
Justin Dirrenberger, Centre des Matériaux P.M. Fourt, Ecole des Mines de Paris, Evry.
Samuel Forest, Centre des Matériaux P.M. Fourt, Ecole des Mines de Paris, Evry.
Anne Françoise Gourgues, Centre des Matériaux P.M. Fourt, Ecole des Mines de Paris, Evry.
Dominique Jeulin, Centre de Morphologie Mathématique, Ecole des Mines de Paris, 35 rue St Honoré, 77300 Fontainebleau.
Christian Lantuejoul, Centre de Géosciences, Ecole des Mines de Paris, 35 rue St Honoré, 77300 Fontainebleau.
Benoit Noetinger, IFP, 78 Rueil Malmaison.
André Pineau, Centre des Matériaux P.M. Fourt, Ecole des Mines de Paris, Evry.
Bruno Sudret, Laboratoire Navier, École des Ponts ParisTech, Champs-sur-Marne.
François Willot, Centre de Morphologie Mathématique, Ecole des Mines de Paris, 35 rue St Honoré, 77300 Fontainebleau.

Organiser: Dominique Jeulin
Email: dominique.jeulin@mines-paristech.fr
Phone: +33 1 64 69 47 06

Time and location: 19–23 March 2012, in Ecole des Mines de Paris (60 Bd Saint-Michel, Paris)
Participants: 30 maximum

Goal:
Many solid media and materials (composites, granular media, metals, biomaterials, porous media, soils, rocks, etc.) encountered in materials sciences, geophysics, environmental sciences, energetics, hydrogeology,... display microstructures and structures of several length scales, showing often a non-deterministic disorder. A better understanding and prediction of the resulting multiscale and random nature of materials' mesoscopic and/or macroscopic properties requires a modeling approach based on a combination of probabilistic concepts with methods of physics and mechanics. The course, which aims to provide an introduction to this subject, will be given in a self-contained series of lectures and training sessions on computers.
First, motivated by a review of advanced experimental techniques for the microstructure description, and by typical results involving fluctuations present in plasticity, damage, fracture, and flows phenomena in porous media, basic tools of applied probability and random processes are recalled. Then, probabilistic tools for the description random media and models together with their simulation are introduced. At the second stage, physics and mechanics of random media are presented from the standpoint of approximate

solutions of partial differential equations with random coefficients. For example, linear electrostatics problems in random media are studied by means of perturbation expansion of the random electric and displacement fields, while bounds on the effective permittivitty and of elastic moduli are derived from variational principles. This approach of homogenization, which can be applied to other physical properties like the composition of permeability, or of the thermal conductivity, is illustrated by third order bounds. The third area of focus concerns the use of numerical techniques (like Finite Elements and FFT based computation), to provide an estimation of homogenized properties of random media from Monte Carlo type simulations. Bounds and numerical techniques are then extended to non linear behaviours, like the plasticity of polycrystals. Given the importance of reliability problems in a multitude of engineering applications, several fracture statistics models (brittle, ductile, fatigue) are worked out from a probabilistic approach.

Structure of the course: Five full days in a single week. Lectures (70%) and practical training on computers (30%).

Course contents (Room to be defined)

Day 1 (Monday March 19th): Introduction and basic concepts:

9h-9h15 General introduction (D. Jeulin)

9h15- 10h30 Introduction to material variability of mechanical properties at different scales (A. Pineau)

10h30-11h Break

11h- 12h30 Introduction to applied probability and probabilistic models (D. Jeulin)

14h-15h Introduction to the simulation of random variables (Ch. Lantuejoul)
15h-16h Advances in experimental techniques: available data at different scales (A.F. Gourgues)

16h-16h30 Break

16h30-18h30 Morphological characterization of random sets and of random functions: size, repartition, connectivity (D. Jeulin)

Day 2 (Tuesday March 20 th): Models and simulation of random media

9h-10h Examples of models and simulation of point processes (Ch. Lantuejoul)

10h-10h30 Examples of models of random sets (Boolean model, dead leaves) and of random functions (Boolean, dead leaves, dilution) (D. Jeulin)

10h30-11h Break

11h-12h30 Examples of models of random sets (Boolean model, dead leaves) and of random functions (Boolean, dead leaves, dilution) (D. Jeulin)

14h-15h Gaussian random functions: properties and (conditional) simulations (Ch. Lantuejoul)
15h-18h Training session on morphological characterization of images and on simulations with the software Micromorph (D. Jeulin) **(Room L027)**

Day 3 (Wednesday March 21rst): Homogenization of random media (linear properties): bounds and numerical techniques

9h-10h Electrostatics of random media: perturbation expansion of the random electrical and displacement fields; estimation of the effective permittivity, statistical approach of the Representative Volume Element (D. Jeulin)
10h-10h40 Classical and Hashin-Shtrikman variational principles; derivation of bounds of effective properties (D. Jeulin)
10h40-11h Break
11h-11h45 Third order bounds of the dielectric permittivity and of the elastic moduli of some models of random media. Examples of optimal microstructures (D. Jeulin)
11h45-12h45 Training session on the calculation of bounds of linear properties of random media (D. Jeulin, J. Dirrenberger) **(Room L027)**

14h- 15h Basis of the homogenization of periodic media. Use of Finite Elements calculation for the homogenization of random media (elasticity and heat conduction). Application to the determination of the RVE (J. Dirrenberger, S. Forest)
15h -16h Numerical homogenization of random media by FFT (F. Willot)
16h – 16h30 Break
16h30-18h30 Training session on Finite Elements (J. Dirrenberger) **(Room L027)**

Day 4 (Thursday March 22nd): Transport in random media. Fracture Statistics, numerical techniques

9h-10h30 Scales and physical properties in porous media (Benoit Noetinger)
10h30-10h45 Break
10h45-12h30 Probabilistic continuum models of brittle fracture (weakest link model, critical damage, crack propagation and arrest, random damage) (D. Jeulin)

14h-18h30 Training session on Finite Elements (J. Dirrenberger) **(Room L027)**

Day 5 (Friday March 23rd): Homogenization of random media (non linear properties), Reliability

9h-10h30 Introduction to non linear constitutive behaviours. Variational methods for non linear composites (Michel Bornert)
10h30-11h Break
11h-12h30 Bounds of non linear mechanical properties. Application to Hashin-Shtrikman, Beran bounds, and to non linear laminates (Michel Bornert)

14h- 16h Introduction to Reliability and to Stochastic Finite Elements (B. Sudret)
16h-16h30 Conclusion

Readings

- M.J. Beran, Statistical Continuum Theories, John Wiley, (1968).
- F.M. Beremin, A local criterion for cleavage fracture of a nuclear pressure vessel steel, Metall. Trans. 14A, 2277, (1983).
- J. Besson, G. Cailletaud, J.L. Chaboche, S. Forest, Mécanique non linéaire des matériaux, Hermes, (2001).
- Th. Bretheau, M. Bornert, P. Gilormini (eds), Homogénéisation en mécanique des matériaux. Vol 1, 2, Hermes, (2001).
- D. Jeulin, (ed.), Advances in Theory and Applications of Random Sets, World Scientific, (1997).
- D. Jeulin, Random structure models for composite media and fracture statistics, in Advances in Mathematical Modelling of Composite Materials, K.Z. Markov (ed.), World Scientific, 239-289, (1994).
- D. Jeulin and M. Ostoja-Starzewski, (ed.), Mechanics of Random and Multiscale Microstructures, CISM Lecture Notes N° 430, Springer Verlag, (2001)
- D. Jeulin, S. Forest (eds) Continuum Models and Discrete Systems CMDS11, Mines Paristech, les Presses, 2008.
- Ch. Lantuéjoul, Geostatistical Simulation. Models and Algorithms, Springer Verlag, (2002).
- G. Matheron, Eléments pour une théorie des milieux poreux, Masson, (1967).
- P. Ponte Castañeda, P. Suquet. "Nonlinear composites." *Advances in Applied Mechanics* 34 (1998): 171-302.
- B. Sudret, Uncertainty propagation and sensitivity analysis in mechanical models - Contributions to structural reliability and stochastic spectral methods. Habilitation à diriger des recherches, Université Blaise Pascal, Clermont-Ferrand (2007). http://bruno.sudret.free.fr/hdr.html
- P. Suquet (ed) : Continuum Micromechanics. CISM Lecture Notes N° 377. Springer-Verlag. 1997.
- J.R. Willis, Variational and related methods for the overall properties of composites, Adv. Appl. Mech. 21, 2-78, (1981).

Prerequisites: Basic knowledge in probability theory, physics and mechanics of solids.
Examination: The students prepare a written project from data processed during the training sessions. The project is submitted 3 weeks after the course